The Fiscal Role of Conscription in the U.S. World War II Effort: Supplementary Material

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This document contains the appendices referred to in the paper.

APPENDIX A

For Table 1, data for total military enrollment and casualties is from the U.S. Department of Veterans Affairs *Fact Sheet: America's Wars*, http://www1.va.gov/opa/fact/amwars.asp. Combat deaths refer to those killed in action or who died of combat wounds. They do not include deaths for other reasons, e.g., due to disease and privation; non-combat deaths were only quantitatively significant for the Civil War. Data for total direct war costs is from Nordhaus (2002). The average annualized GDP share refers to the war cost normalized by GDP over the duration of the war.

The data for adult population corresponds to the total population (including armed forces overseas), 15 years and older, July estimates; these are obtained from the U.S. Census Bureau, www.census.gov/statab/www/minihs.html. Exceptions to this relate only to the calculations of Table 1. For the Civil War, and Persian Gulf War, resident (as opposed to total) population was used. For the Civil War, resident population data are from *Historical Statistics of the United States, Colonial Times to 1970* (U.S. Department of Commerce, 1976), series A7, with imputations by age using series A92-3, A99-100, A120-1 (details on imputations are available from author upon request). *Historical Statistics* is also the source for data on annual active duty military personnel (series Y904), military wage and salary compensation (F167), basic pay plus allowances in the military (D924), and average annual earnings of non-military employees

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(D724) discussed in Section 3.

Total selective service inductions data are available from the U.S. Selective Service System website, www.sss.gov/induct.htm. Data on disaggregated government spending, national income and product accounts, and fixed assets and consumer durable goods are available from the U.S. Department of Commerce, Bureau of Economic Analysis, National Economic Accounts, www.bea.gov/bea/dn1.htm. National income data for the pre-1929 period are from EH.net, eh.net/hmit/gdp/.

Data on federal tax receipts are from the Executive Office of the President (2002). The labor and capital income tax rates correspond to series MTRL1 and MTRK1, respectively from Joines (1981). The market value of total outstanding government debt is the sum of series MPRIV2, MSAVB, and MVSL from Seater (1981). The TFP measures are taken from Kendrick (1961), Appendix A, Table A-XXII, and Christensen and Jorgenson (1995), Table 5.15, column 1. The data for civilian hours worked are from Kendrick (1961), Appendix A, Table A-X. The after-tax real wage data are those displayed as nonfarm compensation per hour in Figure 4 of the 2003 version of McGrattan and Ohanian (2006).

APPENDIX B

The following is the proof of Proposition 1:

Proof. For exposition, denote $h\left(s^t\right)$ as h, and $\varphi\left(s^t\right)$ as $\varphi\left(h\right)$, or simply φ. First, it is obvious that $\varphi\left(h\right)=1$ at $h=\bar{h}$. It remains to show that $\varphi\left(h\right)$ obtains a minimum at $h=\bar{h}$. The first derivative of φ is:

$$\varphi'\left(h\right)=\frac{v''\left(h\right)\left[v\left(h\right)-v\left(\bar{h}\right)\right]}{v'\left(h\right)^2\bar{h}},$$

and the second derivative is:

$$\varphi''\left(h\right)=\frac{v''\left(h\right)v'\left(h\right)^2+\left[v'''\left(h\right)v'\left(h\right)-2v''\left(h\right)^2\right]\left[v\left(h\right)-v\left(\bar{h}\right)\right]}{v'\left(h\right)^3\bar{h}}.$$

As long as $v'\left(h\right)$ is finite, the only critical value for φ is $\varphi'\left(h\right)=0$ at $h=\bar{h}$. Since $v'<0$ and $v''<0$, $\varphi''\left(\bar{h}\right)>0$, so that φ reaches a minimum at $h=\bar{h}$. ■

APPENDIX C

Following the seminal work of Lucas and Stokey (1983), equilibrium in the conscription economy is characterized in primal form. This is a useful first step in deriving results on the optimal implementation of conscription. The primal representation requires consideration of the following two constraints. The first is the aggregate resource constraint which ensures that the private sector output market clears state-by-state:

$$c\left(s^t\right)+k\left(s^t\right)+g\left(s^t\right)=z\left(s^t\right)k\left(s^{t-1}\right)^{\alpha}\left[(1+\gamma)^t h\left(s^t\right)\right]^{1-\alpha}+(1-\delta)\,k\left(s^{t-1}\right),\quad \forall s^t. \tag{1}$$

The second is the implementability constraint which ensures that the government's budget is balanced in present value:

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\left\{u'\left(s^t\right)c\left(s^t\right)+v'\left(s^t\right)\left[\left(1-d\left(s^t\right)\right)h\left(s^t\right)+\phi d\left(s^t\right)\bar{h}\right]\right\}=u'\left(s^0\right)a_0, \tag{2}$$

where $a_0=b_{-1}+\left[r\left(s^0\right)-\theta\left(s^0\right)\left(r\left(s^0\right)-\delta\right)+1-\delta\right]k_{-1}$, and the date-0 rental rate is $r\left(s^0\right)=\alpha z\left(s^0\right)\left[h\left(s^0\right)/k_{-1}\right]^{1-\alpha}$.

Proposition 1 (Appendix) *In any competitive equilibrium, the allocation,* $\{c\left(s^t\right),\ h\left(s^t\right),\ k\left(s^t\right)\}$*, must satisfy constraints (1) and (2). Furthermore, given* ϕ*,* $\theta\left(s^0\right)$*, and sequences* $\{c\left(s^t\right),\ h\left(s^t\right),\ k\left(s^t\right)\}$ *that satisfy these constraints, it is possible to construct all of the remaining equilibrium allocation, price and policy variables.*

Proof. The aggregate resource constraint is obtained easily through substitution. To obtain the implementability constraint take the household's budget constraint, multiply by $\beta^t\pi\left(s^t\right)u'\left(s^t\right)$, and sum over all s^t and t. Using the household's FONCs and the following transversality conditions:

$$\lim_{r\to\infty}\beta^r\pi\left(s^r\right)u\left(s^r\right)k\left(s^r\right)=0,$$

$$\lim_{r\to\infty}\beta^r\pi\left(s^r\right)u'\left(s^r\right)p\left(s^r\right)b\left(s^r\right)=0,$$

for all s^r, this simplifies to obtain (2).

With sequences $\{c\left(s^t\right),\ h\left(s^t\right),\ k\left(s^t\right)\}$ that satisfy (1) and (2), construct the remaining equilibrium objects at s^t as follows. Private sector output, $y\left(s^t\right)$, the rental rate, $r\left(s^t\right)$, and the civilian wage rate, $w\left(s^t\right)$, are given by the firm's production function and FONCs, with

$\tilde{h}\left(s^t\right) = h\left(s^t\right)$ and $\tilde{k}\left(s^t\right) = k\left(s^{t-1}\right)$. Using the household's FONCs, the labor tax rate and the price of a one-period bond are, respectively:

$$\tau\left(s^t\right) = 1 + \frac{v'\left(s^t\right)}{u'\left(s^t\right) w\left(s^t\right)},$$

$$p\left(s^t\right) = \beta \sum_{s^{t+1}|s^t} \pi\left(s^{t+1}|s^t\right) \frac{u'\left(s^{t+1}\right)}{u'\left(s^t\right)}.$$

To obtain real bond holdings, take the household's date r budget constraint, multiply by $\beta^r \pi\left(s^r\right) u'\left(s^r\right)$, and sum over states s^r following s^t for $r \geq t+1$ to get:

$$b\left(s^t\right) = \left[\sum_{r=t+1}^{\infty} \sum_{s^r} \beta^{r-t} \pi\left(s^r|s^t\right) \frac{\chi\left(s^r\right)}{u'\left(s^t\right)} - k\left(s^t\right)\right] / p\left(s^t\right),$$

where $\chi\left(s^r\right) = u'\left(s^r\right) c\left(s^r\right) + v'\left(s^r\right) h\left(s^r\right)\left[\left(1 - d\left(s^r\right)\right) h\left(s^r\right) + \phi d\left(s^r\right) \bar{h}\right]$. Finally, the state s^t capital tax rate:

$$\theta\left(s^t\right) = \left\{g\left(s^t\right) + \phi w\left(s^t\right) d\left(s^t\right) \bar{h} + b\left(s^{t-1}\right) - p\left(s^t\right) b\left(s^t\right) - \tau\left(s^t\right) w\left(s^t\right)\left[\left(1 - d\left(s^t\right)\right) h\left(s^t\right) + \phi d\left(s^t\right) \bar{h}\right]\right\} / \left[\left(r\left(s^t\right) - \delta\right) k\left(s^{t-1}\right)\right],$$

is obtained from the government's budget constraint. ■

Two points deserve mention. First, a similar result to Proposition 1 (Appendix) holds in the case without conscription. In particular, without conscription, the term "$\phi d\left(s^t\right) \bar{h}$" in the implementability constraint, (2), is replaced by the term "$\varphi\left(s^t\right) d\left(s^t\right) \bar{h}$."[1] Second, this economy features a complete set of tax instruments, despite the fact that the government issues non-contingent debt. This can be seen from the primal representation, since the only cross-state restriction on equilibrium allocations is due to the requirement of intertemporal budget balance, (2). In this economy, complete cross-state risk-sharing is achieved through the use of the state-contingent tax rate on capital (see Chari et. al., 1991 and 1994).

In light of Proposition 1 (Appendix), solving for the Ramsey equilibrium is equivalent to finding the allocation $\left\{c\left(s^t\right), h\left(s^t\right), k\left(s^t\right)\right\}$ that maximizes the household's welfare subject to the aggregate resource constraint, (1), and implementability constraint, (2). Let λ denote the Lagrange multiplier associated with (2) and let:

$$W\left(s^t; \lambda\right) \equiv \left[u\left(c\left(s^t\right)\right) + \left(1 - d\left(s^t\right)\right) v\left(h\left(s^t\right)\right) + d\left(s^t\right) v\left(\bar{h}\right)\right] + \lambda\left\{u'\left(s^t\right) c\left(s^t\right) + v'\left(s^t\right)\left[\left(1 - d\left(s^t\right)\right) h\left(s^t\right) + \phi d\left(s^t\right) \bar{h}\right]\right\}.$$

[1] The details of the proof are analogous to that presented here.

The Ramsey problem can be stated as maximizing:

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)W\left(s^t;\lambda\right)-\lambda u'\left(s^0\right)a_0,$$

subject to (1).

In order for this problem to be interesting, it must be that a_0 is sufficiently non-negative. To see this, note that $-a_0$ represents the government's initial asset position (the household's initial liabilities against the government). If $-a_0$ is large, the government could finance its stream of spending by simply running down its assets. In this case the government's intertemporal budget constraint would not bind, $\lambda = 0$, and there would be no need to resort to distortionary taxation or conscription. Hence, attention is restricted to the case where a_0 is sufficiently large, so that $\lambda > 0$. This amounts to restricting the initial values for the capital tax rate, $\theta\left(s^0\right)$, and bond holdings, b_{-1}. Given this characterization, the proof of Proposition 3 is straightforward.

Proof. Let $\mathcal{U}(\phi)$ denote the household's expected lifetime utility in the Ramsey equilibrium for a given value of ϕ. From the envelope condition:

$$\mathcal{U}'(\phi)=\lambda\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)v'\left(s^t\right)d\left(s^t\right)\bar{h}.$$

Since $\lambda > 0$ and $v' < 0$, $\mathcal{U}'(\phi) < 0$. Hence, under conscription, welfare is maximized by minimizing military pay and setting $\phi = 0$. In the case without conscription, the proportionality factor ϕ is replaced by $\varphi\left(s^t\right)$. But since $\varphi\left(s^t\right) \geq 1$ (see Proposition 1), welfare in this case is always lower than in the case with conscription. ■

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