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Optimal fiscal and monetary policy with sticky prices ☆

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Abstract

In this paper I consider the role of state-contingent inflation as a fiscal shock absorber in an economy with nominal rigidities. I study the Ramsey equilibrium in a monetary model with distortionary taxation, nominal non-state-contingent debt, and sticky prices. With sticky prices, the Ramsey planner must balance the shock absorbing benefits of state-contingent inflation against the associated resource misallocation costs. For government spending processes resembling post-war experience, introducing sticky prices generates striking departures in optimal policy from the case with flexible prices. For even small degrees of price rigidity, optimal policy displays very little volatility in inflation. Tax rates display greater volatility compared to the model with flexible prices. With sticky prices, tax rates and real government debt exhibit behavior similar to a random walk. For government spending processes resembling periods of intermittent war and peace, optimal policy displays extreme inflation volatility even when the degree of price rigidity is large. As the variability in government spending increases, smoothing tax distortions across states of nature becomes increasingly important, and the shock absorber role of inflation is accentuated.

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1. Introduction

An important result of the optimal policy literature is the prescription of policies which smooth tax distortions over time and across states of nature. When governments finance stochastic government spending by taxing labor income and issuing one-period debt, state-contingent returns on that debt allow tax rates to be roughly constant (see Lucas and Stokey, 1983; Chari et al., 1991). In monetary models, this tax smoothing can be achieved even when nominal returns on debt are not state-contingent; varying the price level in response to shocks allows the government to achieve appropriate state-contingent, ex-post real returns (see Chari et al., 1991). Generating inflation in the period of a positive spending shock allows the government to decrease its real liabilities by reducing the value of its outstanding nominal claims. In this way, the government is able to attenuate the increase in taxes required to maintain present value budget balance. Similarly, a deflation in response to a negative spending shock attenuates the required fall in tax revenues.

Clearly, inflation plays an important policy role when nominal returns to debt are not state-contingent, since it can generate real returns which are.[1,2] A quantitative property of these models is that when calibrated to post-war US data, optimal policy displays extreme inflation volatility (see Chari et al., 1991; Chari and Kehoe, 1999). This is due to the fact that inflation is costless in these models.

The aim of this paper is to determine the optimal degree of volatility when inflation is no longer costless, but still has shock absorbing benefits. This is an important consideration since studies that consider optimal monetary policy devoid of fiscal considerations prescribe stable inflation when nominal rigidities are present (see King and Wolman, 1999; Erceg et al., 2000; Khan et al., 2000).

To study this question I introduce sticky prices into the standard cash–credit good model. When some prices in the economy are set before the realization of government spending, unanticipated inflation causes relative price distortions. This distortion generates costly misallocation of real resources. Optimal policy on the part of the government must balance the tax smoothing benefits of state-contingent inflation against these misallocation costs.

To see this trade-off, consider an economy with a complete set of tax instruments, as in Lucas and Stokey (1983) or Chari et al. (1991). With both sticky price and flexible price firms, this can be achieved by providing the government with an additional state-contingent tax on the output of sticky price firms. In this complete instruments case, ex-post variation in the price level generates state-contingency in the real value of government liabilities, keeping tax distortions smooth. At the same

[1] Sims (2001) extends this analysis to address the debate on dollarization. Replacing debt denominated in domestic currency with debt denominated in foreign currency eliminates the feasibility of state-contingent returns generated through inflation.

[2] For discussion of these results in relation to the literature on the Fiscal Theory of the Price Level, see Woodford (1998) and Christiano and Fitzgerald (2000). These papers, as well as Sims (2001), leave as an open question the optimal degree of inflation volatility when inflation is costly.

time, ex-post variation in the tax on the output of sticky price firms keeps the relative price, and hence relative production, across sticky and flexible price firms at its efficient level. In the model presented in this paper, the government does not levy independent taxes on the output of sticky and flexible price firms so that the complete instruments outcome cannot be attained. Ex-post inflation drives down the relative price of sticky price firms, leading to overproduction of sticky price goods (and underproduction of flexible price goods). Likewise, ex-post deflation leads to underproduction (overproduction) of sticky price (flexible price) goods. Hence, any variation in ex-post prices used to generate real state-contingency in nominal debt results in misallocation of resources.

In examples calibrated to post-war US data, I show that introducing sticky prices has a striking impact on the optimal degree of inflation volatility. While the flexible price model displays extreme volatility, the analogous sticky price model displays essentially stable deflation at the rate of time preference. This is true even when the proportion of sticky price setters is small. For instance, when 2% of price setting firms post prices before the realization of shocks, the standard deviation of inflation falls by a factor of 8 (relative to the case with flexible prices); when 5% of firms have sticky prices, the standard deviation falls by a factor of 16. When the model displays diminishing marginal product of labor, 5% sticky prices causes the standard deviation of inflation to fall by a factor of 40. Hence, for post-war calibrations, the gains from achieving state-contingency in real debt returns are small relative to the misallocation costs induced by variable ex-post inflation.

The nominal interest rate is no longer zero in the sticky price model, as prescribed by Friedman (1969), but instead fluctuates across states of nature. However, the deviation from the Friedman Rule is quantitatively small. Finally, the serial correlation properties of optimal tax rates and real government debt differ markedly in the two environments. In contrast to Barro's (1979) random walk result, Chari et al. (1991) show that with flexible prices, these variables inherit the serial correlation of the model's underlying shocks (see Lucas and Stokey, 1983, for the initial exposition of this result with state-contingent debt). Faced with sticky prices, a benevolent government finances increased spending largely through increased taxes. As high spending regimes persist, tax revenues are gradually increased and real debt is accumulated. During spells of low spending, taxes fall and accumulated debt is paid off. As a result, the autocorrelations of these objects are near unity regardless of the persistence in the shock process, partially reviving Barro's result. This finding is similar to that of Aiyagari et al. (2002) who consider optimal policy in a model with incomplete markets (i.e. real non-state-contingent debt). In fact, I show that the sequence of restrictions imposed on the set of feasible equilibria by sticky prices and market incompleteness are analytically similar.

As the volatility in government spending increases, the shock absorbing benefits of state-contingent inflation come to dominate the costs of resource misallocation. For instance, when government spending is 3 times more volatile than post-war experience, optimal policy prescribes extreme volatility in inflation even with a large proportion of sticky prices. With large spending shocks, tax smoothing considerations are accentuated, and the Ramsey planner tolerates misallocation associated with volatile inflation in order to smooth tax distortions across states of nature. To shed light on this result, I analyze the nature of the benefits and costs of inflation volatility when government spending is volatile.

The next section presents a cash–credit good model with price setting on the part of intermediate good firms; a subset of these firms post prices before the realization of the state of nature. Section 3 characterizes equilibrium, and develops the primal representation of equilibrium. I show that the primal representation requires consideration of two sequences of *cross-state* constraints not present with flexible prices. Section 4 presents the Ramsey allocation problem. The existence of the cross-state constraints makes solving this problem difficult. Section 5 provides additional analysis of the key cross-state constraint introduced by sticky prices. Section 6 discusses the characteristics of the Ramsey equilibrium that make development of a solution method feasible. I show that the solution builds upon the recursive contracts approach developed in Marcet and Marimon (1999). Section 7 presents quantitative results. Section 8 concludes.

2. The model

Let s_t denote the event realization at any date t, where $t = 0, 1, \ldots$. The history of date-events realized up to date t is given by the history, or *state*, $s^t = (s_0, s_1, \ldots, s_t)$. The unconditional probability of observing state s^t is denoted $\pi(s^t)$, while the probability of observing s^t given state s^{t-1} is denoted $\pi(s^t|s^{t-1}) \equiv \pi(s^t)/\pi(s^{t-1})$. The initial state, s^0, is given so that $\pi(s^0) = 1$.

The economy is populated by a large number of atomistic households and final good producing firms, a continuum of intermediate good producing firms, and a government (a combined fiscal and monetary authority). Each of these agents is described in turn.

2.1. Households

There is a large number of identical, infinitely lived households in the economy. The representative household's objective function is

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(c_1(s^t), c_2(s^t), l(s^t)), \quad \beta \in (0, 1). \tag{1}$$

The period utility function is of the form:

$$U(c_1, c_2, l) = \left[\left\{ \left[(1-\gamma)c_1^{\phi} + \gamma c_2^{\phi} \right]^{1/\phi} (1-l)^{\psi} \right\}^{1-\sigma} - 1 \right] / (1-\sigma), \tag{2}$$

with $\sigma, \psi > 0$, $\phi < 1$, and $0 < \gamma < 1$. Here, l denotes the share of the household's unit time endowment devoted to labor, c_1 denotes units of consumption good purchased in cash, and c_2 denotes consumption purchased on credit.

The household faces two sequences of constraints. The first is the flow budget constraint which must hold for all s^t. This is relevant during securities trading in each period; trading occurs after observing the current realization of s_t:

$$M(s^t) + B(s^t) \leqslant R(s^{t-1})B(s^{t-1}) + (1 - \tau(s^{t-1}))I(s^{t-1}) + M(s^{t-1}) - P(s^{t-1})c_1(s^{t-1}) - P(s^{t-1})c_2(s^{t-1}). \tag{3}$$

Holdings of cash chosen at s^t are denoted $M(s^t)$. Holdings of nominal debt chosen at s^t are denoted $B(s^t)$, and earn a return of $R(s^t)$ upon maturity at date $t+1$. Nominal wealth carried from state s^{t-1} to date t (the right-hand side of the inequality) derives from bond income, after-tax production income, and excess cash holdings, less consumption purchases made on credit from the previous period.

Nominal production income at state s^t (payable at the beginning of date $t+1$) derives from wage payments, $W(s^t)l(s^t)$, and profits earned from intermediate goods producers:

$$I(s^t) = W(s^t)l(s^t) + \int_0^1 \Pi_i(s^t)\,\mathrm{d}i. \tag{4}$$

Here, $\Pi_i(s^t)$ denotes intermediate good firm i's profit, for $i \in [0, 1]$. Finally, $\tau(s^t)$ is a uniform, distortionary income tax rate levied on both dividend and labor income.

After securities trading, the household supplies labor, $l(s^t)$, at the nominal wage $W(s^t)$, and buys consumption, $c_1(s^t)$ and $c_2(s^t)$, at the price $P(s^t)$. Purchases of the cash good are subject to a cash-in-advance constraint:

$$P(s^t)c_1(s^t) \leqslant M(s^t), \quad \forall s^t. \tag{5}$$

State s^t purchases made in cash are settled at s^t, while purchases made on credit are settled at the beginning of date $t+1$.

This generates the standard first-order necessary conditions (FONCs) for the household:

$$-\frac{U_l(s^t)}{U_2(s^t)} = (1 - \tau(s^t))\frac{W(s^t)}{P(s^t)}, \tag{6}$$

$$\frac{U_1(s^t)}{U_2(s^t)} = R(s^t), \tag{7}$$

$$\frac{1}{R(s^t)} = \frac{\beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t)U_1(s^{t+1})/P(s^{t+1})}{U_1(s^t)/P(s^t)}. \tag{8}$$

Here and in the rest of the paper, $U_1 = \partial U/\partial c_1$ (similarly for c_2), and $U_l = \partial U/\partial l$. Eq. (6) states that the presence of a non-zero tax rate drives a wedge between the marginal rate of substitution in leisure-consumption and the real wage. Eq. (7) states that a non-zero nominal interest rate drives a wedge between the marginal rate of substitution in cash–credit good consumption and the marginal rate of transformation, which is unity (see Eq. (11) below). Eq. (8) states the standard pricing formula for a risk-free (non-state-contingent) nominal bond.

Again, note that the bond return is non-state-contingent: the return at date t is independent of the event realization, s_t.[3] As a result, nominal wealth carried from state s^{t-1} to date t is independent of the realization, s_t, so that:

$$M(\underline{s}^t) + B(\underline{s}^t) = M(\bar{s}^t) + B(\bar{s}^t), \tag{9}$$

for all $\underline{s}^t$, $\bar{s}^t$ following s^{t-1}. Of course, the household's real wealth varies across date t realizations, depending on the value of $P(s^t)$.

2.2. Final good firms

Firms in the final good sector transform intermediate goods into output according to

$$Y = \left[\int_0^1 Y_i^{1/\mu}\,\mathrm{d}i\right]^\mu, \quad \mu > 1, \tag{10}$$

where Y is final output and Y_i is input purchased from intermediate good firm i. Final goods are transformed linearly into household and government consumption, so that

$$c_1(s^t) + c_2(s^t) + g(s^t) \leqslant Y(s^t), \quad \forall s^t. \tag{11}$$

Intermediate good firms have monopoly power over their particular good i. At the end of each period, before the realization of next period's shock, a fraction, $v \in [0, 1]$, of these firms must post their prices for next period; these are the sticky price firms. Flexible price firms post prices after observing the shock. In a symmetric equilibrium:

$$Y(s^t) = [v Y_\mathrm{s}(s^t)^{1/\mu} + (1-v) Y_\mathrm{f}(s^t)^{1/\mu}]^\mu, \tag{12}$$

where s stands for 'sticky' and f stands for 'flexible'.

The market for final goods is perfectly competitive and output is sold at the price $P(s^t)$.[4] The representative final good firm's problem is to choose inputs to maximize profits:

$$P(s^t) Y(s^t) - v P_\mathrm{s}(s^{t-1}) Y_\mathrm{s}(s^t) - (1-v) P_\mathrm{f}(s^t) Y_\mathrm{f}(s^t), \tag{13}$$

when type s and f firms act symmetrically. Technology displays constant returns to scale and equilibrium profits in this sector equal zero. Maximization produces the following FONCs:

$$\frac{P_\mathrm{s}(s^{t-1})}{P(s^t)} = \left(\frac{Y(s^t)}{Y_\mathrm{s}(s^t)}\right)^{(\mu-1)/\mu}, \tag{14}$$

[3] This is also true of cash holdings which earn zero interest in all states.

[4] Payment is received both in the form of cash at period t (on sales of c_1), and cash at the beginning of period $t+1$ (on sales of c_2 and g). Since no interest is earned on cash held 'overnight', the law of one price holds.

$$\frac{P_{\mathrm{f}}(s^t)}{P(s^t)} = \left(\frac{Y(s^t)}{Y_{\mathrm{f}}(s^t)}\right)^{(\mu-1)/\mu}. \tag{15}$$

In accordance with the information structure, the sticky price firm's price at state s^t, $P_{\mathrm{s}}(s^{t-1})$, is a function of the history s^{t-1} only, and is identical across realizations of s_t; as a result:

$$P(\underline{s}^t)\left(\frac{Y(\underline{s}^t)}{Y_{\mathrm{s}}(\underline{s}^t)}\right)^{(\mu-1)/\mu} = P(\bar{s}^t)\left(\frac{Y(\bar{s}^t)}{Y_{\mathrm{s}}(\bar{s}^t)}\right)^{(\mu-1)/\mu}, \tag{16}$$

for all $\underline{s}^t$, $\bar{s}^t$ following s^{t-1}. The value of $Y_{\mathrm{s}}(s^t)$ differs across date t realizations since demand depends on the relative price, $P_{\mathrm{s}}(s^{t-1})/P(s^t)$.

2.3. Intermediate good firms

Each intermediate good firm $i \in [0, 1]$ produces a differentiated good according to

$$Y_i = L_i^{\alpha}, \quad \alpha \leqslant 1. \tag{17}$$

Labor is hired from a perfectly competitive labor market at the nominal wage rate W. Considering $\alpha < 1$ allows for an additional degree of curvature in studying the distortions due to asymmetry in prices and, consequently, asymmetry in labor allocations across flexible and sticky price firms.

2.3.1. Flexible price firms

After observing the current state, s^t, the representative flexible price firm sets its price to maximize profit:

$$\Pi_{\mathrm{f}}(s^t) = P_{\mathrm{f}}(s^t)Y_{\mathrm{f}}(s^t) - W(s^t)L_{\mathrm{f}}(s^t), \tag{18}$$

taking the final good firm's demand, (15), as given. The FONC for this problem:

$$P_{\mathrm{f}}(s^t) = \frac{\mu}{\alpha} W(s^t)L_{\mathrm{f}}(s^t)^{1-\alpha}, \tag{19}$$

states the familiar condition that labor is hired up to the point where the nominal wage is equal to a fraction, $1/\mu$, of its marginal revenue product.

2.3.2. Sticky price firms

Before observing s_t, the representative sticky price firm's problem is to choose a price, $P_{\mathrm{s}}(s^{t-1})$, identical across states s^t following s^{t-1}, to maximize:

$$\sum_{s^t|s^{t-1}} \pi(s^t|s^{t-1})Q(s^t)[P_{\mathrm{s}}(s^{t-1})Y_{\mathrm{s}}(s^t) - W(s^t)L_{\mathrm{s}}(s^t)]. \tag{20}$$

The term in square brackets is the firm's state s^t profit, $\Pi_{\mathrm{s}}(s^t)$. The sticky price firm takes the marginal value of dividends, $Q(s^t) = (1-\tau(s^t))U_2(s^t)/P(s^t)$, and the final good firm's demand, (14), as given. The FONC for this problem reads

$$\sum_{s^t|s^{t-1}} \pi(s^t|s^{t-1})Q(s^t)P(s^t)^{\mu/(\mu-1)}Y(s^t)\left[P_s(s^{t-1}) - \frac{\mu}{\alpha}W(s^t)L_s(s^t)^{1-\alpha}\right] = 0. \quad (21)$$

2.4. Government

Government consumption, $g(s^t)$, is determined exogenously and transits between $\{\underline{g}, \bar{g}\}$ with symmetric transition probability $\rho \in (0, 1)$. The government purchases $g(s^t)$ on credit, and faces a flow budget constraint:

$$M(s^t) + B(s^t) \geqslant M(s^{t-1}) + R(s^{t-1})B(s^{t-1}) + P(s^{t-1})g(s^{t-1}) - \tau(s^{t-1})I(s^{t-1}). \quad (22)$$

This must be satisfied for all s^t through the appropriate choice of taxation, nominal non-state-contingent debt issue, and inflation (via money creation) which induces ex-post variation in the real value of outstanding liabilities.[5]

3. Characterizing equilibrium

An equilibrium in which intermediate good firms of each type behave symmetrically is defined in the usual way.

Definition 1. Given the household's initial real claims on the government, $a_0 > 0$, and the stochastic process, $\{g(s^t)\}$, a *symmetric equilibrium* is an allocation, $\{c_1(s^t),\ c_2(s^t),\ l(s^t),\ L_f(s^t),\ L_s(s^t),\ Y(s^t),\ Y_f(s^t),\ Y_s(s^t),\ B(s^t)\}$, price system, $\{R(s^t),\ P(s^t),\ P_f(s^t),\ P_s(s^{t-1}),\ W(s^t)\}$, and government policy, $\{M(s^t),\ \tau(s^t)\}$, such that:

- $\{c_1(s^t),\ c_2(s^t),\ l(s^t),\ M(s^t),\ B(s^t)\}$ solves the household's problem subject to the sequence of household budget constraints and cash-in-advance constraints;
- $\{Y(s^t),\ Y_f(s^t),\ Y_s(s^t)\}$ solves the final good firm's problem;
- $\{P_f(s^t),\ L_f(s^t),\ Y_f(s^t)\}$ solves the flexible price firm's problem;
- $P_s(s^{t-1})$ solves the sticky price firm's problem (with $L_s(s^t)$ and $Y_s(s^t)$ being demand determined);
- the sequence of government budget constraints is satisfied;
- the labor market clears:

$$l(s^t) = vL_s(s^t) + (1-v)L_f(s^t), \quad \forall s^t; \quad (23)$$

- and $R(s^t) \geqslant 1,\ \forall s^t$.

[5] Note that the role of inflation in generating state-contingent returns can be alleviated by providing the government with additional tax instruments. Possibilities include explicit state-contingent debt, non-state-contingent debt with a sufficiently rich maturity structure (see Angeletos, 2002; Buera and Nicolini, 2001), capital taxation (see Chari et al., 1991), or consumption taxation (see Correia et al., 2003). Chari and Kehoe (1999) provide further discussion.

The final condition ensures that the consumer does not find it profitable to buy money and sell bonds, so that the cash-in-advance constraint holds with equality.[6] Bond market clearing at each state has been implicitly assumed, as both issues and holdings are denoted by the single variable, $B(s^t)$; the same is true of money, $M(s^t)$. Clearing in the market for each intermediate good i has also been implicitly assumed, with purchases and production denoted by $Y_i(s^t)$. Clearing in the final goods market is satisfied by Walras' Law.[7]

Finally, note that it is initial real, as opposed to nominal, claims that are taken as given. This ensures that the initial price level, $P(s^0)$, cannot be used by the government to generate zero real indebtedness or arbitrarily large revenues at date 0. As such, $P(s^0)$ is normalized to unity. This has the additional consequence of ensuring that $P_s(s^{-1}) = P(s^0)$, so that the results across flexible and sticky price models are not driven simply by the treatment of date 0.

3.1. The primal approach

To simplify the analysis of optimal policy, I adopt the approach of characterizing equilibrium in primal form. This involves restating the equilibrium conditions in terms of real allocations alone. In Proposition 2, I show that equilibrium imposes five constraints on the allocation for consumption and labor. As well, given consumption and labor that satisfy these constraints, it is possible to recover the equilibrium values for the remaining allocation, price, and policy variables.

The first two primal form constraints guarantee that $R(s^t) \geqslant 1$ and that the final goods market clears:

$$U_1(s^t) \geqslant U_2(s^t), \tag{24}$$

$$c_1(s^t) + c_2(s^t) + g(s^t) = F(s^t), \tag{25}$$

where $F(s^t) = [vL_s(s^t)^{\alpha/\mu} + (1-v)L_f(s^t)^{\alpha/\mu}]^{\mu}$. These must hold $\forall s^t$. Call these the *no arbitrage* and *aggregate resource constraints*.

The third constraint ensures that the government's budget is balanced in present value:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) C(s^t) = U_1(s^0) a_0. \tag{26}$$

Here, $C(s^t) = U_1(s^t)c_1(s^t) + U_2(s^t)c_2(s^t) + U_l(s^t)\Lambda(s^t)$ is the utility value of the government's real budget surplus at s^t (this is shown below), and $\Lambda(s^t) = (\mu/\alpha)[(1-v)L_f(s^t) + vL_f(s^t)^{1-(\alpha/\mu)}L_s(s^t)^{\alpha/\mu}]$. Eq. (26) is the standard *implementability constraint* modified to account for: (i) monopolistic competition in intermediate goods and (ii) asymmetry between flexible and sticky price firms. The fourth constraint is a

[6] When $R(s^t) = 1$, the return to money and bonds are equal and the assets are redundant. To resolve this, I adopt the convention that the cash-in-advance constraint is binding in these states.

[7] This can be verified by combining the two budget constraints (holding with equality), labor market clearing, and the final good firm's FONCs.

rewriting of the *sticky price firm's FONC* in terms of real allocations:

$$\sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) U_l(s^{t+1}) h(s^{t+1}) = 0, \quad \forall s^t, \tag{27}$$

where $h(s^{t+1}) = L_{\rm f}(s^{t+1})^{1-(\alpha/\mu)} L_{\rm s}(s^{t+1})^{\alpha/\mu} - L_{\rm s}(s^{t+1})$.

The final constraint is the *sticky price constraint*. This condition ensures that $P_{\rm s}(s^t)$ is identical across realizations of s_{t+1}. Given that government spending takes on two possible values, $\underline{g}$ and $\bar{g}$, this constraint is

$$\sum_{s^{t+1}|s^t} d(s^{t+1}) A(s^{t+1}) \sum_{r=1}^{\infty} \sum_{s^{t+r}|s^{t+1}} \beta^r \pi(s^{t+r}|s^{t+1}) \frac{C(s^{t+r})}{U_1(s^{t+1})} = 0, \quad \forall s^t, \tag{28}$$

where

$$d(s^{t+1}) = \begin{cases} -1, & \text{if } g_{t+1} = \underline{g}, \\ +1, & \text{if } g_{t+1} = \bar{g}, \end{cases} \tag{29}$$

$$A(s^{t+1}) = \left[(1-v) \left(\frac{L_{\rm s}(s^{t+1})}{L_{\rm f}(s^{t+1})} \right)^{-\alpha/\mu} + v \right]^{1-\mu}. \tag{30}$$

Proposition 2. *In any symmetric equilibrium, the allocation for consumption and labor,* $\{c_1(s^t),\ c_2(s^t),\ L_{\rm f}(s^t),\ L_{\rm s}(s^t)\}$, *must satisfy the five constraints,* (24)–(28). *Furthermore, given sequences* $\{c_1(s^t),\ c_2(s^t),\ L_{\rm f}(s^t),\ L_{\rm s}(s^t)\}$ *that satisfy these constraints, it is possible to construct all of the remaining equilibrium real allocation, price and policy variables.*

Proof. Denote the real wage and real bond holdings by $w(s^t) \equiv W(s^t)/P(s^t)$ and $b(s^t) \equiv B(s^t)/P(s^t)$, respectively. I first show that any equilibrium allocation must satisfy (24)–(28). Deriving (24) and (25) requires only straightforward substitution. Since the government's budget constraint and (25) together imply that the household's budget constraint holds with equality, to derive the implementability constraint start with the household's date t budget constraint. Multiply this by $\beta^t \pi(s^t) U_1(s^t)/P(s^t)$, and sum over all s^t and t. Using the transversality condition:

$$\lim_{r \to \infty} \beta^r \pi(s^r) U_1(s^r) b(s^r) = 0, \quad \forall s^r, \tag{31}$$

together with (7), (8), and (5), this simplifies to

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \{ U_1(s^t) c_1(s^t) + U_2(s^t)[c_2(s^t) - n(s^t)] \} = U_1(s^0) a_0. \tag{32}$$

Here, $n(s^t) = (1-\tau(s^t))[(1-v)\Pi_{\rm f}(s^t) + v\Pi_{\rm s}(s^t) + W(s^t) l(s^t)]/P(s^t)$ is real after-tax production income at s^t. Using (15), (14), (19), and (23), $n(s^t) = \Lambda(s^t)(1-\tau(s^t)) w(s^t)$. Using (6) and substituting above obtains (26).

To get the sticky price firm's FONC in terms of real allocations, substitute in (6) to obtain

$$\sum_{s^t|s^{t-1}} \pi(s^t|s^{t-1})U_l(s^t)\frac{P(s^t)^{\mu/(\mu-1)}Y(s^t)}{W(s^t)}\left[P_s(s^{t-1}) - \frac{\mu}{\alpha}W(s^t)L_s(s^t)^{1-\alpha}\right] = 0. \tag{33}$$

Using (15), (14), (19), and multiplying by $P_s(s^{t-1})^{\mu/(1-\mu)}$ (a constant across states s^t following s^{t-1}), results in expression (27).

The sticky price constraint ensures that the price $P_s(s^{t-1})$ is identical across states $\bar{s}^t$ and $\underline{s}^t$ following s^{t-1}, as stated in (16). From the household's date t budget constraint:

$$\frac{P(s^t)}{P(s^{t-1})} = \frac{R(s^{t-1})b(s^{t-1}) + (1-\tau(s^{t-1}))Y(s^{t-1}) - c_2(s^{t-1})}{c_1(s^t) + b(s^t)}. \tag{34}$$

Substitute (34) into (16) and divide by $P(s^{t-1})$ to obtain

$$[c_1(\bar{s}^t) + b(\bar{s}^t)]\left(\frac{L_s(\bar{s}^t)^{\alpha}}{Y(\bar{s}^t)}\right)^{(\mu-1)/\mu} = [c_1(\underline{s}^t) + b(\underline{s}^t)]\left(\frac{L_s(\underline{s}^t)^{\alpha}}{Y(\underline{s}^t)}\right)^{(\mu-1)/\mu}. \tag{35}$$

Take the household's date r budget constraint, multiply by $\beta^r\pi(s^r)U_1(s^r)/P(s^r)$, and sum over states s^r following s^t for $r \geqslant t+1$ to get

$$b(s^t) = \sum_{r=t+1}^{\infty}\sum_{s^r|s^{t+1}} \beta^{r-t}\pi(s^r|s^t)\frac{C(s^r)}{U_1(s^t)} + \frac{U_2(s^t)}{U_1(s^t)}c_2(s^t) + \frac{U_l(s^t)}{U_1(s^t)}\Lambda(s^t). \tag{36}$$

To obtain (28), substitute this into (35) for $b(\bar{s}^t)$ and $b(\underline{s}^t)$.

Given sequences $\{c_1(s^t),\ c_2(s^t),\ L_f(s^t),\ L_s(s^t)\}$ that satisfy (24)–(28), the remaining equilibrium variables are constructed as follows. Real balances at s^t are given by $M(s^t)/P(s^t) = c_1(s^t)$. The gross risk-free rate of return is $R(s^t) = U_1(s^t)/U_2(s^t)$. Aggregate output is $Y(s^t) = F(s^t)$. Relative prices are given by (14) and (15). From (19) the real wage is

$$w(s^t) = \frac{\alpha}{\mu}\frac{P_f(s^t)}{P(s^t)}L_f(s^t)^{\alpha-1}. \tag{37}$$

From (6) the tax rate is

$$\tau(s^t) = 1 + \frac{U_l(s^t)}{U_2(s^t)w(s^t)}. \tag{38}$$

Real government debt at s^t is given by (36). Finally, the gross rate of inflation between states s^t and s^{t-1} is given by (34). □

For models in which the government has access to a complete set of tax instruments, the primal form is characterized only by constraints (24)–(26) (with $L_f(s^t) = L_s(s^t),\ \forall s^t$). This is true when the government is able to issue explicit state-contingent debt (see Lucas and Stokey, 1983), or when inflation acts costlessly to render nominal non-state-contingent debt state-contingent in real terms (see Chari et al., 1991; and Appendix A). In the face of stochastic shocks, tax distortions can be smoothed across states of nature in these models. However, sticky prices moves policy away from this complete instruments result. The presence of sticky prices adds to (24)–(26) the infinite sequence of *cross-state* constraints, (28)—one at each state, s^t—to the set of feasible equilibria.[8] The implications of this for the nature of optimal tax smoothing are discussed in Section 5.

4. The Ramsey problem

The Ramsey planner's problem is the following: find the fiscal and monetary policy that induces symmetric equilibrium associated with the highest value of the household's expected lifetime utility. Call this equilibrium the *Ramsey equilibrium*. Specifically, the government commits to its chosen policy at the beginning of time; in all periods, maximizing agents behave taking this policy plan as given.[9]

In light of Proposition 2, solving for the Ramsey equilibrium is equivalent to finding the consumption and labor allocation that maximizes the household's utility subject to constraints (24)–(28). Let $\{\delta(s^t)\}$, $\{\theta(s^t)\}$, λ, $\{\eta(s^t)\}$, and $\{\xi(s^t)\}$, respectively, be the multipliers associated with these constraints in the Lagrange formulation of the problem. For $t \geqslant 1$, the FONCs for consumption are:

$$\begin{aligned} & U_j(s^t) + \delta(s^t)[U_{1j}(s^t) - U_{2j}(s^t)] + \eta(s^{t-1})U_{lj}(s^t)h(s^t) \\ & + \left[\lambda + \sum_{r=0}^{t-1} \xi(s^r)\tilde{d}(s^{r+1})\frac{A(s^{r+1})}{U_1(s^{r+1})}\right] C_j(s^t) \\ & - \xi(s^{t-1})\tilde{d}(s^t)\frac{A(s^t)U_{1j}(s^t)}{[U_1(s^t)]^2} q(s^t) = \theta(s^t), \end{aligned} \tag{39}$$

for $j = 1, 2$. Here, $C_j = \partial C/\partial c_j$, $\tilde{d}(s^t) = d(s^t)/\pi(s^t|s^{t-1})$ and $q(s^t) = \sum_{r=0}^{\infty}\sum_{s^{t+r}|s^t}\beta^r\pi(s^{t+r}|s^t)C(s^{t+r})$ is the present (utility) value of real government budget surpluses from state s^t onward (this is shown below). The FONC for $L_s(s^t)$ is

$$\begin{aligned} & vU_l(s^t) + \delta(s^t)v[U_{1l}(s^t) - U_{2l}(s^t)] + \eta(s^{t-1})[vU_{ll}(s^t)h(s^t) \\ & + U_l(s^t)h_s(s^t)] + \left[\lambda + \sum_{r=0}^{t-1} \xi(s^r)\tilde{d}(s^{r+1})\frac{A(s^{r+1})}{U_1(s^{r+1})}\right] C_s(s^t) \\ & + \xi(s^{t-1})\tilde{d}(s^t)\frac{A_s(s^t)}{U_1(s^t)} q(s^t) = -\theta(s^t)F_s(s^t). \end{aligned} \tag{40}$$

Here, $h_s = \partial h/\partial L_s$ and similarly for the functions C, A, and F. An analogous FONC holds for $L_f(s^t)$.

The complete instruments case drops constraints (27) and (28) from the primal representation, so that the terms involving $\{\eta(s^t)\}$ and $\{\xi(s^t)\}$ are omitted from the Ramsey problem and its associated FONCs. As a result, the implementability

[8] Price rigidity also introduces the infinite sequence of constraints, (27), to the primal form. However, without sticky prices, (27) would hold trivially in any symmetric equilibrium, with $L_f = L_s$.

[9] For discussion of time consistency issues and the relationship to Stackelberg equilibrium see Lucas and Stokey (1983), Chari et al. (1995), and Woodford (1998).

constraint provides the sole cross-state link, so that the FONCs depend only on state s^t variables and the value of λ. With sticky prices, the entire *infinite history*, s^∞, matters for optimal s^t decisions due to the inclusion of the two sequences of cross-state restrictions, (27) and (28). Further discussion is presented in Section 6.

4.1. Optimality of the Friedman Rule

In the rest of this section, I present results on the optimality of the Friedman Rule. In particular, for the class of utility functions (2), it is possible to show the following:

Proposition 3. *For the imperfectly competitive, cash–credit good model with sticky prices, the Friedman Rule is not optimal; that is,* $R(s^t) = 1$ *does not hold* $\forall s^t$ *in the Ramsey equilibrium.*

Proof. There are two cases in which $R(s^t) \equiv U_1(s^t)/U_2(s^t) = 1$ holds in the Ramsey equilibrium: (a) $U_1(s^t) = U_2(s^t)$ is 'unconstrained optimal', or (b) $\delta(s^t) > 0$ and $U_1(s^t) = U_2(s^t)$ is 'constrained optimal'. I first show that case (a) cannot hold at any s^t, $t \geqslant 1$. Suppose $U_1(s^t) = U_2(s^t)$ is unconstrained optimal. Drop constraint (24) and equate the FONCs of the Lagrange problem with respect to $c_1(s^t)$ and $c_2(s^t)$, $t \geqslant 1$. Since $U_{1l}/U_1 = U_{2l}/U_2$ and $C_1/U_1 = C_2/U_2$ for preferences satisfying (2), this simplifies to

$$\xi(s^{t-1})\tilde{d}(s^t)\frac{A(s^t)}{U_1(s^t)}q(s^t)[U_{11}(s^t) - U_{12}(s^t)] = 0, \tag{41}$$

where $q(s^t) > 0$. Since the sticky price constraint, (28), is binding in the Ramsey equilibrium, $\xi(s^t) \neq 0$. Hence, (41) holds iff $U_{11}(s^t) = U_{12}(s^t)$; this is a contradiction since $U_{11}(s^t) < U_{12}(s^t)$ for $U_1(s^t) = U_2(s^t)$ and preferences satisfying (2). Therefore, case (a) cannot hold at any s^t, $t \geqslant 1$.

It remains to show that case (b) cannot hold for all s^t. Suppose $\delta(s^t) > 0$ and $U_1(s^t) = U_2(s^t)$, for all s^t, $t \geqslant 1$. Equate the FONCs of the Lagrangian with respect to $c_1(s^t)$ and $c_2(s^t)$, $t \geqslant 1$, to get:

$$\delta(s^t)U_1(s^t)\left[1 + \frac{c_1(s^t)}{c_2(s^t)}\right] = \xi(s^{t-1})\tilde{d}(s^t)\frac{A(s^t)}{U_1(s^t)}q(s^t). \tag{42}$$

Since $\xi(s^{t-1})$ is constant across states s^t following s^{t-1}, and $d(s^t)$ fluctuates between $\{-1, +1\}$, $\delta(s^t) > 0$ is contradicted. Hence, case (b) cannot hold for all s^t, and therefore, the Friedman Rule is not optimal. $\square$

This stands in contrast to the result for the analogous flexible price model (in which all date t prices are set after observing the realization of s_t). In this case, optimality of the Friedman Rule holds for a more general class of utility functions, which includes (2). In Appendix A, I analyze the flexible price model and prove the following:

Proposition 4. *Let preferences be homothetic in cash and credit good consumption, and weakly separable in leisure. Then, for the imperfectly competitive, cash–credit good model with flexible prices, the Friedman Rule is optimal.*

Evidently, non-optimality of the Friedman Rule stems from the presence of sticky prices, and in particular, the sticky price constraint, (28), which restricts the set of feasible equilibria relative to the (complete instruments) case with flexible prices. Further discussion of this is contained in the following section. Also, note that this result differs from that emphasized in Schmitt–Grohé and Uribe (2001), who consider an imperfectly competitive monetary model with flexible prices. In their model, the Friedman Rule is not optimal—even *without* sticky prices—due to an assumption that profit income is untaxed. Indeed, if I modify the flexible price, cash–credit good economy studied in Appendix A so that profit income goes untaxed, the Friedman Rule breaks down as well. Further discussion of this result, as well as its relationship to the 'uniform commodity taxation rule' is contained in Appendix A.

5. The sticky price constraint

The introduction of sticky prices causes the optimality of the Friedman Rule to break down. Price rigidity also causes Ramsey tax rates and real bond holdings to display a higher degree of persistence than that of the underlying shocks. Both of these features are better understood upon closer inspection of the sticky price constraint, (28). This constraint requires that the present value of real government budget surpluses be equated across states $\underline{s}^t$ and $\bar{s}^t$ following s^{t-1}, up to the factor $A(\underline{s}^t)/A(\bar{s}^t)$. That is:

Proposition 5. *For states $\underline{s}^t$ and $\bar{s}^t$ following s^{t-1}, $t \geqslant 1$, the sticky price constraint can be restated as*

$$A(\underline{s}^t)\mathscr{PV}(\underline{s}^t) = A(\bar{s}^t)\mathscr{PV}(\bar{s}^t), \tag{43}$$

where

$$\mathscr{PV}(s^t) = \sum_{r=t}^{\infty} \sum_{s^r|s^t} \beta^{r-t}\pi(s^r|s^t)\frac{U_1(s^r)}{U_1(s^t)}\left[\frac{\tau(s^r)Y(s^r) - g(s^r)}{R(s^r)} + \frac{M(s^r)}{P(s^r)}\left(\frac{R(s^r)-1}{R(s^r)}\right)\right]. \tag{44}$$

See Appendix B for the derivation. In the expression for $\mathscr{PV}(s^t)$, the first term in square brackets is the government's real primary budget surplus at s^r, $r \geqslant t$, adjusted for the timing on spending and tax revenue in the government's budget constraint. The second term is the real interest savings the government earns from issuing money relative to debt. Evidently, $\mathscr{PV}(s^t)$ is the present value of government surpluses (from all sources) from state s^t onward.

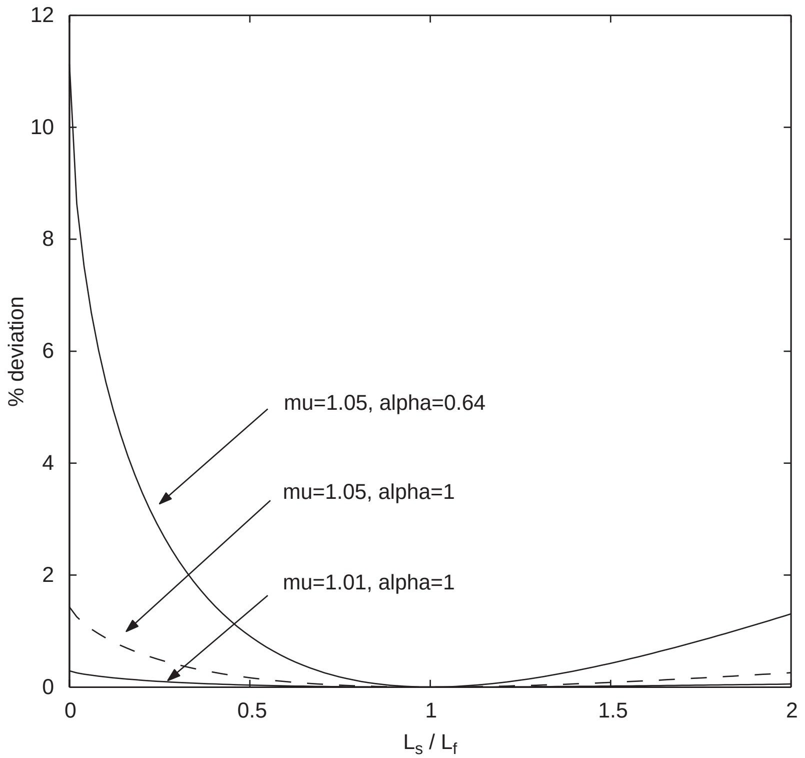

Fig. 1. Resource misallocation costs. Measured as percentage deviation in actual output from latent output due to asymmetry in labor across flexible and sticky price firms.

This constraint summarizes the trade-off between cross-state tax smoothing and resource misallocation faced by the Ramsey planner. To see this, first note that in any equilibrium, the government's real liabilities at s^t must be exactly equal to the present value stream of real surpluses earned from s^t onward. Second, the term $A(s^t)$ depends on the degree of resource misallocation at state s^t. To see this, consider the term:

$$\mathcal{O} = 1 - \left[v\left(\frac{L_s}{L_f}\right)^{\alpha/\mu} + 1 - v\right]^{\mu} \Big/ \left[v\left(\frac{L_s}{L_f}\right) + 1 - v\right]^{\alpha}, \tag{45}$$

which measures the percentage deviation of actual output from 'latent' output, $[vL_s + (1-v)L_f]^{\alpha}$, due to asymmetry in labor across sticky and flexible price firms. This is plotted in Fig. 1 for $v = 0.25$.[10] This measure of output loss is zero when $L_s/L_f = 1$, and positive whenever the ratio L_s/L_f deviates from unity. Note also that

[10] The plot of the misallocation cost function is asymmetric for two reasons: (1) the endpoints of the plotted range represent disproportionate deviations of L_s/L_f from unity (at $L_s/L_f = 2$, L_s is 100% greater than L_f, while at $L_s/L_f = 0$, L_s is infinitely smaller than L_f), and (2) $v \neq 0.5$. Misallocation costs are in fact symmetric for proportionate deviations from unity when $v = 0.5$.

increasing μ and decreasing α accentuates the concavity of final goods production (as a function of either L_s or L_f), resulting in larger misallocation costs. Hence, deviations in $A(s^t)$ from unity represent periods in which misallocation costs are non-zero. For states following s^{t-1}, the sticky price constraint requires that the real value of government liabilities be equalized up to the factor $A(\underline{s}^t)/A(\bar{s}^t)$.

This cross-state restriction is obviously absent from the flexible price model. Indeed, Chari and Kehoe (1999) show that with flexible prices, optimality entails $\mathscr{PV}(\underline{s}^t) > \mathscr{PV}(\bar{s}^t)$.[11] Given that nominal government liabilities inherited at date t are equal across states (see Eq. (9)), a jump in the price level in the high spending state drives down the real value of liabilities, $\mathscr{PV}(\bar{s}^t)$, relative to $\mathscr{PV}(\underline{s}^t)$ in the low spending state. This allows tax distortions to be smoothed across states of nature while maintaining government budget balance.

With sticky prices, constraint (43) states that this use of state-contingent inflation cannot be achieved costlessly. In situations where it is optimal to use inflation for tax smoothing purposes (so that $\mathscr{PV}(\underline{s}^t) > \mathscr{PV}(\bar{s}^t)$), the Ramsey planner must tolerate resource misallocation, represented by deviations of $A(\underline{s}^t)/A(\bar{s}^t)$ from unity. In situations where the costs of misallocation dominate the benefits of cross-state tax smoothing, labor allocations across sticky and flexible price firms are close to symmetric. This is manifested as $A(s^t) \simeq 1$ and $\mathscr{PV}(\underline{s}^t) \simeq \mathscr{PV}(\bar{s}^t)$.

In the latter case, the sticky price constraint can be interpreted as an approximation to the constraint found in the following model: a real economy (without money or sticky prices) in which state-contingent returns on debt are explicitly ruled out. This is exactly the environment considered in Aiyagari et al. (2002), who partially revive the intertemporal tax smoothing result of Barro (1979). With non-state-contingent real returns, the government's real liabilities at date t are determined one period in advance. Hence, when the model of Aiyagari et al. (2002) is simplified so that government spending takes on only two values, the sequence of constraints imposed by incomplete contingent claims markets is

$$\mathscr{PV}(\underline{s}^t) = \mathscr{PV}(\bar{s}^t), \tag{46}$$

for states $\underline{s}^t$ and $\bar{s}^t$ following s^{t-1}. For a derivation of this, see Appendix B. In the sticky price model with $A(s^t) \simeq 1$, $\mathscr{PV}(\underline{s}^t) \simeq \mathscr{PV}(\bar{s}^t)$. Hence, the restrictions on the set of feasible equilibria imposed by sticky prices and incomplete markets are approximately equivalent. In both environments the government lacks a complete set of tax instruments, and tax smoothing across states of nature is replaced with tax smoothing over time; this is manifested in the persistence properties of tax rates and real government debt.

Finally, to better understand Proposition 3, consider a deviation from the Friedman Rule. Raising the nominal interest rate from zero has two first-order effects on $\mathscr{PV}(s^t)$: it decreases the real value of the time-adjusted primary surplus and increases the real value of interest savings. Since the primary surplus is orders of magnitudes greater than interest savings, an increase in the nominal rate decreases

[11] This is true even in the case of i.i.d. shocks, since $\mathscr{PV}(s^t)$ includes the current period surplus.

the present value of surpluses.[12] Hence with sticky prices, the Ramsey planner uses a positive nominal interest rate during periods of low spending to decrease $\mathscr{PV}(\underline{s}^t)$ and help satisfy the sticky price constraint. This alleviates the need to use state-contingent inflation which results in resource misallocation.

6. A recursive solution method

As shown in Appendix A, equilibrium with flexible prices can be characterized as allocations that satisfy constraints, (24)–(26). As a result, optimal consumption and labor allocations at state s^t are stationary functions of only the current realization of government spending, g_t, and the value of λ associated with the implementability constraint, (26). This is because the only cross-state effect of the allocation at s^t is through its effect on the current utility value of the real government budget surplus, $C(s^t)$, in maintaining present value budget balance. This makes the flexible price model particularly tractable, and exact solutions can be found (see Chari and Kehoe, 1999; and Schmitt–Grohé and Uribe, 2001 for discussion).

This stationarity result does not hold for the sticky price model due to the presence of the additional cross-state constraints, (27) and (28). The appearance of *future* $C(s^{t+r})$ terms—for all s^{t+r} following s^t, $r \geqslant 1$—in the *current* state s^t sticky price constraint makes the problem difficult to solve. As a consequence, the Ramsey planner's s^t choice of allocation, and the resulting value of $C(s^t)$, affects not only the implementability constraint, but also all sticky price constraints along the history up to s^t. This can be seen in the Ramsey planner's FONCs, (39) and (40).

To understand this, consider an increase in $\mathscr{PV}(s^r)$, the present value of real surpluses earned from s^r onward. From the state s^{r-1} sticky price constraint, the value of this perturbation in terms of lifetime utility is given by $\beta^r\pi(s^{r-1})\xi(s^{r-1})d(s^r)A(s^r)$.[13] Note, however, that the utility value of the real surplus earned at s^t, $C(s^t)$, impacts every present value term along the history leading to s^t. As a result, a perturbation in $C(s^t)$ impacts every sticky price constraint along the history s^t; for any state s^r leading to s^t (with $r < t$), the value of a perturbation in $C(s^t)$ is given by

$$\beta^t\pi(s^t)\frac{\xi(s^{r-1})d(s^r)A(s^r)}{\pi(s^r|s^{r-1})U_1(s^r)}. \tag{47}$$

[12] For the baseline sticky price model presented below, the average simulated value of the primary surplus is 1800 times greater than that of interest savings.

[13] Recall that optimal policy with flexible prices involves $\mathscr{PV}(\underline{s}^r) > \mathscr{PV}(\bar{s}^r)$ so that the sticky price constraint is strictly binding. Hence, the value of an increase in $\mathscr{PV}(s^r)$ is negative when $g_r = \underline{g}$, since this exacerbates satisfying the sticky price constraint. Conversely, when $g_r = \bar{g}$ a positive perturbation of $\mathscr{PV}(s^r)$ is beneficial so that $\beta^r\pi(s^{t-1})\xi(s^{r-1})d(s^r)A(s^r)$ is positive. These results, together with the definition of $d(s^r)$, imply that $\xi(s^r) > 0$ for all s^r.

To account for this, as well as the effect of $C(s^t)$ on the implementability constraint, define the pseudo-state variable:

$$\kappa(s^{t-1}) = \lambda + \sum_{r=0}^{t-1} \xi(s^{r-1})\tilde{d}(s^r)\frac{A(s^r)}{U_1(s^r)}, \quad t \geqslant 2, \tag{48}$$

where $\tilde{d}(s^r) = d(s^r)/\pi(s^r|s^{r-1})$. Since the initial state, s^0, is given, $\kappa(s^{t-1}) = \lambda$ for $t = 0, 1$. This variable summarizes the influence of $C(s^t)$ on the implementability constraint and the sequence of sticky price constraints along the history s^t, and evolves according to the law of motion:

$$\kappa(s^t) = \kappa(s^{t-1}) + \xi(s^{t-1})\tilde{d}(s^t)A(s^t), \quad t \geqslant 1. \tag{49}$$

As discussed in Marcet and Marimon (1999), this pseudo-state variable captures the impact of past events on the choice of current allocations.

An additional consequence of the sticky price constraint is that future surpluses impacts upon current allocations. To this end, I write the present utility value of real budget surpluses from s^t onward as $q(s^t)$:

$$q(s^t) = \sum_{r=0}^{\infty} \sum_{s^{t+r}|s^t} \beta^r \pi(s^{t+r}|s^t)C(s^{t+r}). \tag{50}$$

For $t \geqslant 1$, this function is defined recursively as

$$q(s^t) = C(s^t) + \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t)q(s^{t+1}). \tag{51}$$

This summarizes the impact of future decision variables upon decisions at the current state.

These definitions allow for a recursive representation of the Ramsey problem. Specifically, optimal allocations for $t \geqslant 1$ are stationary in the state $(\kappa(s^{t-1}), g_t \mid g_{t-1})$. Accordingly, denote:

- $(g_t \mid g_{t-1})$ by Γ, where $\Gamma \in \{(\underline{g}|\underline{g}), (\bar{g}|\underline{g}), (\underline{g}|\bar{g}), (\bar{g}|\bar{g})\}$, and
- $\kappa(s^{t-1})$ by κ, where $\kappa \in \mathscr{R}$.

For the sake of exposition, I continue to use the '$|$' relation to denote the timing of shocks; therefore, $(\bar{g}|\underline{g})$ represents '$\bar{g}$ at date t following $\underline{g}$ at date $t-1$'. Hence, allocations such as $c_1(s^t)$ and $L_f(s^t)$, as well as the present value $q(s^t)$, are stationary functions denoted $c_1(\kappa, \Gamma)$, $L_f(\kappa, \Gamma)$, and $q(\kappa, \Gamma)$. The multipliers on the cross-state constraints, (27) and (28), are stationary functions in $(\kappa(s^{t-1}), g_{t-1})$; that is, $\eta(s^{t-1}) = \eta(\kappa, g_{-1})$ and similarly for $\xi(s^{t-1})$, where $g_{-1} \in \{\underline{g}, \bar{g}\}$ denotes the realization of government spending at date $t-1$. Finally, the pseudo-state variable evolves according to

$$\kappa' = \kappa + \xi(\kappa, g_{-1})\tilde{d}(\Gamma)A(\kappa, \Gamma). \tag{52}$$

Dependence of real variables (such as real debt holdings) and policy variables (such as tax rates) on κ imparts a persistent component to these objects.

Chari et al. (1995) show that inference on the quantitative properties of policies and prices are sensitive to the choice of solution method. In particular, they find that non-linear approximations provide important accuracy improvements relative to linearization methods. Moreover, the presence of the occasionally binding constraint, $R(s^t) \geqslant 1$, is problematic for standard linearization techniques. As a result, I develop a non-linear method to solve the Ramsey problem. Details of the solution algorithm are presented in an appendix to Siu (2002). Key to the technique is finding a non-linear approximation to the function:

$$q(\kappa, \Gamma) = C(\kappa, \Gamma) + \beta \sum_{\Gamma'} \pi(\Gamma') q(\kappa', \Gamma'), \quad \forall(\kappa, \Gamma), \tag{53}$$

which characterizes the Ramsey equilibrium. This is done using the projection methods described in Judd (1992a). Because the solution technique does not rely on remaining within a neighborhood of the model's steady-state, I am also able to study optimal policy for both small and large government spending shocks.

7. Quantitative results

In this section I present results illustrating the quantitative properties of the Ramsey equilibrium with sticky prices. As a benchmark, I also present results for the analogous flexible price model.

7.1. Parameterization

The values of β and σ are set to 0.97 and 1.25, respectively, which are typical values for models calibrated to annual data (see for instance, Chari et al., 1991; Jones et al., 2000). The elasticity and share parameters in consumption are set to $\phi = 0.79$ and $\gamma = 0.62$; these are estimated from the money demand regression described in Chari et al. (1991) upon data from 1960:I to 1999:IV. The value of ψ is set so that in steady-state, 30% of the household's time is spent working.

For the baseline case, the value of μ is set to 1.05. This is somewhat smaller than that used in other quantitative sticky price models, but is closer to the estimates of Basu and Fernald (1997).[14] Since the degree of resource misallocation is increasing in μ (see Fig. 1), this was chosen conservatively to ensure that the baseline parameterization does not overstate the case for inflation stability. For similar reasons, the value of labor's share of income, α, is initially set to unity. The steady-state ratio of government spending to GDP is 20%. The persistence parameter, $\rho = 0.95$, and the values of $\bar{g}$ and $\underline{g}$ are chosen to match the autocorrelation and 6.7% standard deviation of annual US data from 1960 to 1999.

Initial real claims on the government are set so that in the stationary equilibrium of the flexible price model, the government's real debt to GDP ratio is 0.75. This is somewhat larger than the value considered in Chari et al. (1991); it is also larger than

[14] Chari et al. (2000) and Khan et al. (2000) consider $\mu = 1.11$.

the ratio of privately held government debt to GDP observed in the US, which averaged 0.41 from 1960 to 1999, and 0.45 since the Reagan administration.[15] However, choosing a_0 to match this statistic would underestimate the true value of nominal liabilities held against the government by the US private sector. For instance, Judd (1992b) argues that the stock of unused capital depreciation allowance incorporated in the tax code is valued between 25% and 33% of GDP.[16] In addition, the real value of non-indexed government expenditures such as wage and transfer payments depends on the ex-post realization of inflation.[17] The value chosen here is an estimate based on these considerations. Finally, in the sticky price model the fraction of sticky price firms for the baseline case is set at 5%.

7.2. Volatility

Simulation results for the baseline flexible and sticky price models are reported as case (a) of Table 1, where all rates are expressed as annual percentages. From Proposition 4, the Friedman Rule is optimal with flexible prices. With sticky prices, despite the breakdown of the Friedman Rule, optimal nominal interest rates are still close to zero. The value of the interest rate depends largely on the realization of government spending, Γ. When current spending is high, irrespective of the previous spending shock (or the value of the pseudo-state variable), the interest rate is constrained by the 0% lower bound. When current spending is low, the interest rate is positive. The interest rate has a mean of 0.20% in continuation states, $(\underline{g}|\underline{g})$, and attains its largest values in transition states $(\underline{g}|\bar{g})$, where the maximum simulated value is 3.57%.

More striking is the impact on the volatility of the income tax and inflation rates due to the introduction of sticky prices. When the fraction of sticky price firms increases from 0% to 5%, the standard deviation of the Ramsey tax rate increases by a factor of 17; the standard deviation of the inflation rate falls by a factor of 16. At 5% sticky prices, the volatility of inflation is remarkably small. If the inflation rate was normally distributed, 90% of observations would lie between −3.48% and −2.19%. The analogous interval for the flexible price model is bounded by −12.74% and 7.56%.[18]

In fact, optimal inflation volatility is small even when the degree of price rigidity is less than that displayed in Table 1. Fig. 2 plots the standard deviation of the Ramsey inflation rate on the left-scale (solid line) at various values of v. Inflation volatility

[15] The corresponding averages for the ratio of privately held federal government debt to GDP are 0.31 and 0.37.

[16] See Judd (1992b) also for a discussion on the effects of unanticipated inflation on real tax liabilities due to the differential treatment of corporate and personal income tax liabilities.

[17] To obtain a gauge on the value of these current liabilities, note that the ratio of government wages to GDP is approximately 0.10, and the corresponding ratio for Social Security payments is approximately 0.04.

[18] During the completion this paper, I have learned of independent work by Schmitt–Grohé and Uribe (2002) who present similar results in a model where costs of inflation are imposed as a quadratic cost of price adjustment.

Table 1
Quantitative properties with flexible and sticky prices

Rate (annual %)	(a) Baseline		(b) $\alpha = 0.64$		(c) $\mu = 1.01$	
	Flexible	5% Sticky	Flexible	5% Sticky	Flexible	5% Sticky
Nominal interest						
Mean	0	0.162	0	0.133	0	[a]
Std. deviation	N/A	0.509	N/A	0.439	N/A	[a]
Autocorrelation	N/A	0.021	N/A	0.019	N/A	[a]
Income tax						
Mean	22.44	24.44	22.53	24.40	22.53	22.57
Std. deviation	0.068	1.164	0.078	1.226	0.062	0.063
Autocorrelation	0.888	0.984	0.888	0.991	0.888	0.889
Inflation						
Mean	−2.589	−2.835	−2.568	−2.878	−2.607	−2.658
Std. deviation	5.075	0.323	5.368	0.135	4.941	4.903
Autocorrelation	−0.010	0.327	−0.013	0.330	−0.011	−0.011
Money growth						
Mean	−2.599	−2.793	−2.587	−2.850	−2.617	−2.667
Std. deviation	4.841	3.058	4.946	2.467	4.709	4.685
Autocorrelation	−0.007	−0.512	−0.007	−0.481	−0.007	−0.007

[a] In case (c), the deviations of the nominal interest rate from zero are in the seventh decimal place, so these results are not reported.

falls quickly as the fraction of sticky price firms increases. With 2% sticky prices, the standard deviation of inflation is only 0.66%, 8 times smaller than in the flexible price case. The right-scale plots the standard deviation of the Ramsey tax rate (dashed line). With 2% sticky prices, the standard deviation of taxes is 1.07%, 16 times greater than in the case with flexible prices.

The difference in inflation volatility is more dramatic with a greater degree of curvature in production. From Fig. 1, when α decreases from 1 to 0.64, deviations in the ratio L_s/L_f from unity result in larger costs of misallocation. This is seen in case (b) of Table 1 where $\alpha = 0.64$ (and all other parameters as in the baseline case). Evidently, the planner's incentive to reduce misallocation costs is strengthened; with sticky prices the standard deviation of the tax rate increases by a factor of 16 relative to the case with flexible prices, and the standard deviation of the inflation rate falls by a factor of 40. Hence, a benevolent government is faced with a strong incentive to eliminate the resource misallocation that arises from ex-post inflation, relative to the distortions due to variability in tax rates across states of nature.

To further illustrate this trade-off between tax smoothing and resource misallocation, consider the extreme case in which final production exhibits perfect substitution across intermediate goods. With $\mu = 1$ there are no costs associated with asymmetric labor allocation, so that the complete instruments result re-emerges with sticky prices. Quantitatively, with sufficiently small μ and government spending

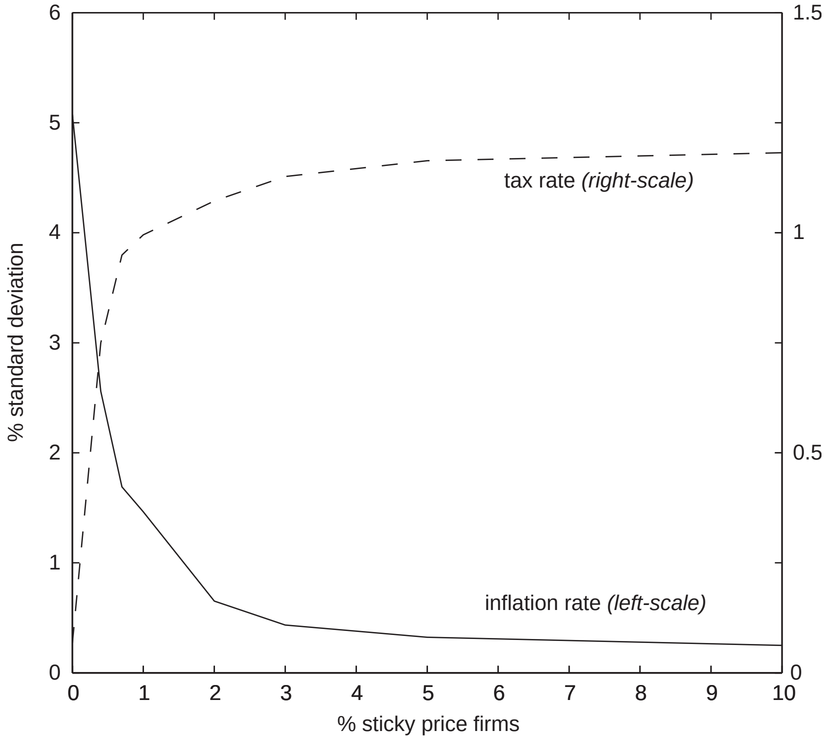

Fig. 2. Optimal inflation and tax rate volatility. Measured as percentage standard deviation for various degrees of price rigidity.

shocks calibrated to match post-war data, the tax smoothing benefits of state-contingent inflation dominate the costs of misallocation. This is illustrated in case (c), where $\mu = 1.01$ (and all other parameters as in the baseline case), so that the elasticity of substitution across intermediate goods is equal to 101; in this case, the quantitative difference between the flexible and sticky price models is minimal.

The final experiment in this subsection investigates the effect of a_0 on optimal inflation volatility. Fig. 3 plots the standard deviation of the Ramsey inflation rate for various values of the real debt to output ratio (all other parameters as in the baseline case); this is done for both the flexible price (dashed lines) and sticky price (solid lines) models. As the liabilities base increases, the government is able to generate the same change in real claims with smaller variations in the price level (for further discussion see Sims, 2001). As a result, the inflation volatility required to achieve cross-state tax smoothing in the flexible price model falls from a standard deviation of 8.90 to 2.72% as the debt to output ratio rises from 0.40 to 1.50. Hence, the extreme degree of inflation volatility found in Chari et al. (1991) with flexible prices is in part due to their parameterization of a_0. With sticky prices, the result of essentially stable inflation is robust across values of a_0.

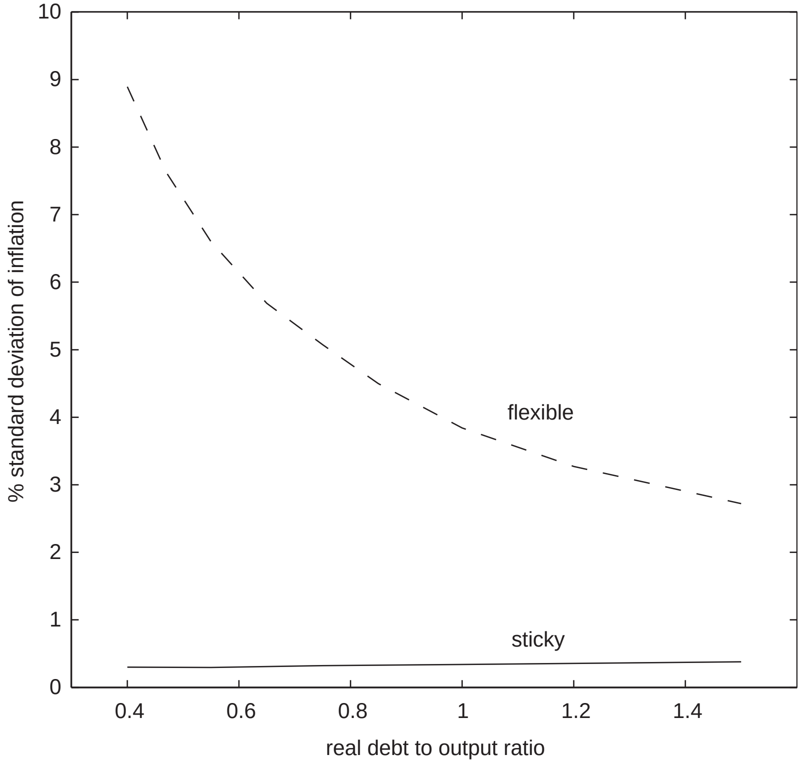

Fig. 3. Optimal inflation volatility as a function of the real liabilities base.

7.3. Time series realizations

With a jump in spending, the government's present value of current and future real liabilities increases. With flexible prices, the government finances this principally through a large devaluation of its outstanding liabilities by generating an inflation. This is not true with sticky prices. This is illustrated in Fig. 4, where I display 25-period time series of simulated data generated from the flexible price (dashed lines) and sticky price (solid lines) models for the baseline parameterization.

In period 5, government spending transits from its low to high state; real spending falls at date 20. With flexible prices, the government responds contemporaneously to an increase in spending by generating a large inflation. In panel B, the inflation rate jumps from -3.93% at date 4 to 20.8% at date 5. This sharply reduces the real value of inherited liabilities, as seen in panel C. Real outstanding liabilities fall by 21% in the period of the shock, and a further 3% in the following period (when payment on date 5 spending is made) due principally to a reduction in real bond issues in period 5. This allows the government to keep the tax rate (panel D) essentially constant across low and high spending states. The tax rate increases only 0.14% in period 5. When spending falls in period 20, there is a deflation and the value of real liabilities rises. Again taxes move very little.

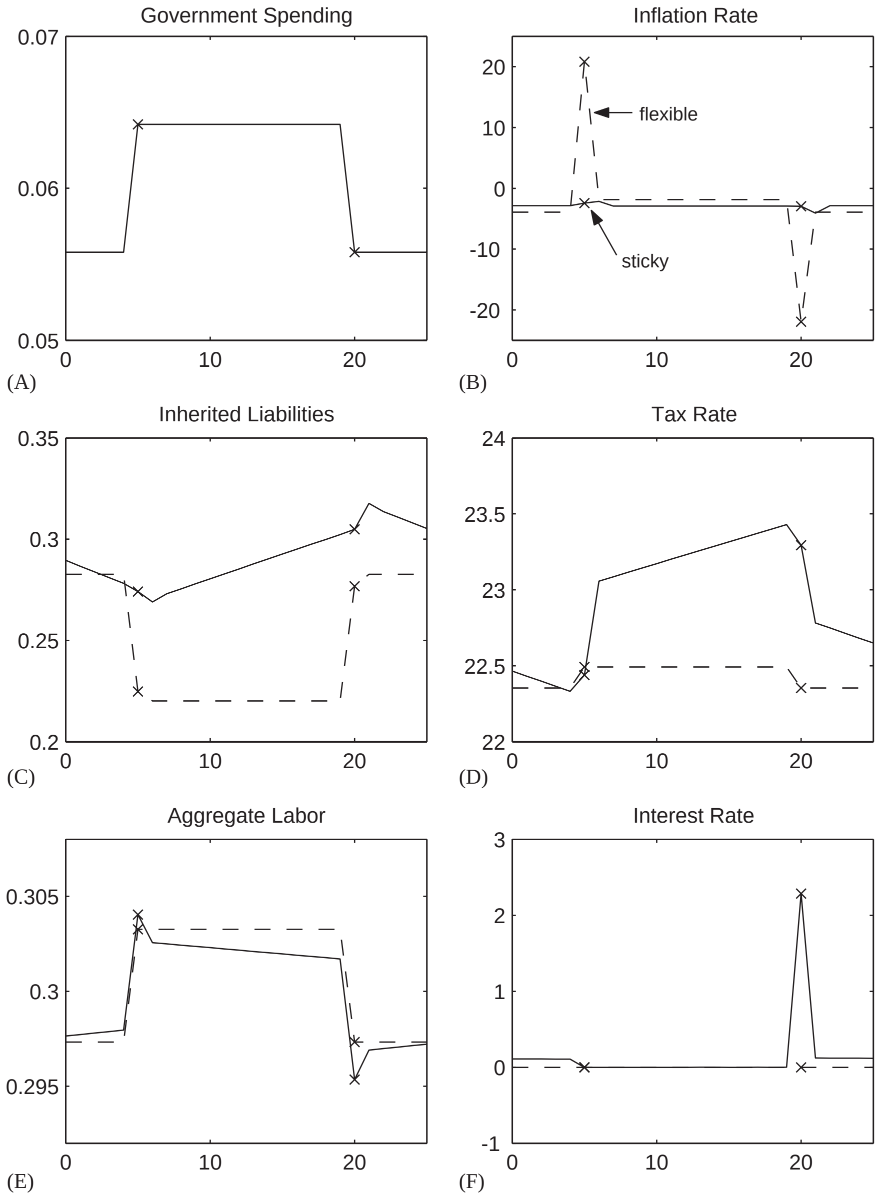

Fig. 4. Simulated time series. Model responses to a government spending shock with flexible (dashed lines) and sticky prices (solid lines).

In the sticky price model, there is essentially no inflationary response to the spending shock. The inflation rate increases from -2.87% in period 4 to -2.43% in period 5, and again to -2.16% in period 6 when payment on the increased spending is due. As a result, there is a much smaller reduction in the real value of inherited liabilities, which falls by only 1.5% in period 5 and 2.1% in period 6. Instead, the government finances the spending shock largely through increased tax revenue and, as the high spending regime persists, through bond issue. Between periods 4 and 20, the tax rate increases 1.11%; during this time, real debt issue (not shown) increases by 14.4%. When government spending falls, tax revenues are lowered, and the government gradually pays down the accumulated debt (with a one period lag). Hence, with sticky prices optimal policy can be roughly characterized as smoothing tax distortions intertemporally; this is accomplished by issuing and retiring debt in response to fiscal shocks. The similarity of this policy to the prescription advocated by Barro (1979) is discussed below.

7.4. Persistence

With sticky prices, the serial correlation of the Ramsey tax rate exhibits a noticeable deviation from the case with flexible prices. With flexible prices, the simulated tax rate inherits the autocorrelation of government spending (see also Lucas and Stokey, 1983; Chari et al., 1991). In the sticky price model, the autocorrelation is much closer to unity. This is also true of real government debt.

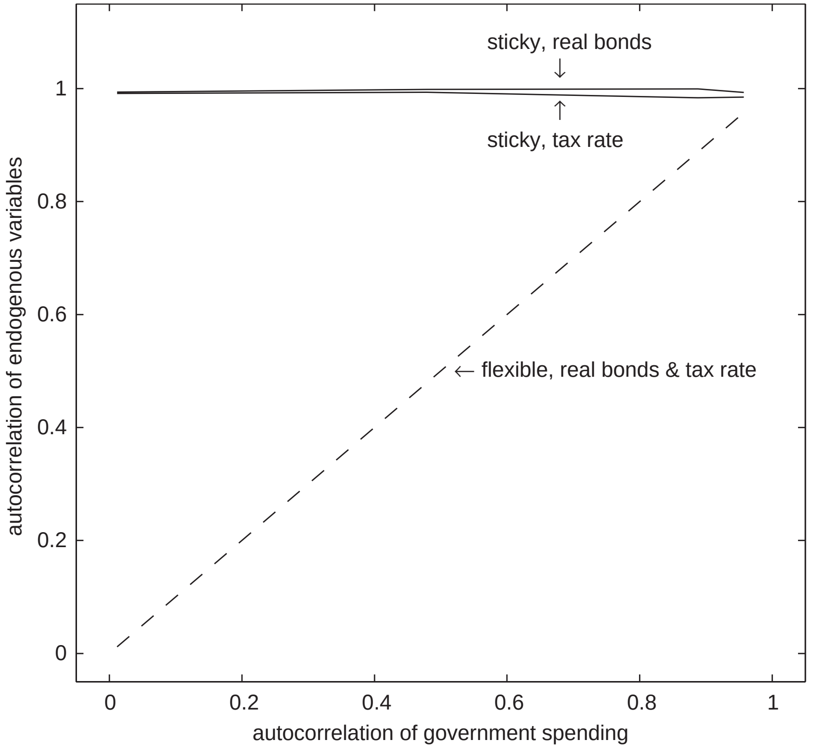

Fig. 5. Persistence in the flexible and sticky price models.

Fig. 5 plots the autocorrelation of tax rates and real bond holdings for various values of the autocorrelation in government spending (all other parameters as in the baseline case). For the flexible price model (dashed line), this relationship maps out the 45° line due to the stationarity result discussed in Section 6; that is, since real variables at state s^t vary only with the value of g_t, the tax rate and bond holdings inherit the serial correlation of the shock process. With sticky prices (solid lines), the autocorrelation of these variables is near unity regardless of the autocorrelation in government spending. The dependence of the tax rate and bond holdings on the pseudo-state, κ, as well as the unit coefficient on its lagged value in the law of motion (52), imparts a persistent component to these variables. In this sense, introducing price rigidity moves optimal policy towards Barro's (1979) random walk result. As in Aiyagari et al. (2002), this behavior is driven by the introduction of a pseudo-state variable summarizing the history of cross-state constraints. In the present model, this pseudo-state summarizes the history of sticky price constraints (28).

7.5. Large shocks

The results presented above can be interpreted as follows: for government spending processes resembling post-war experience, smoothing tax distortions over time represents, in welfare terms, a close substitute to smoothing tax distortions across states of nature. Comparing simulated utility between the baseline flexible and sticky price models confirms this; in order to make the household indifferent between the two economies, consumption in the sticky price model must be increased by only 0.21% in all periods. Hence, for even small degrees of price rigidity, the Ramsey planner is willing to forego state-contingent returns obtained through inflation in order to minimize allocation distortions.

However, when differences in government spending across states are magnified, the value of inflation as a fiscal shock absorber is accentuated. Fig. 6 presents the optimal degree of inflation volatility as a function of v, for government spending with a 21% standard deviation (all other parameters as in the baseline case). This represents variability in spending roughly 3 times that of post-war US experience, so that this exercise can be loosely interpreted as representing periods of war and peace. As shown in Fig. 6, inflation remains volatile even when half of all firms have sticky prices. Over this range, optimal inflation volatility falls very little, from 14.7% to 12.6%. Hence, for sufficiently large spending shocks, tax smoothing considerations dominate and optimal policy prescribes extreme inflation volatility even when the fraction of sticky price firms is large. To better understand this, it is fruitful to consider the costs and benefits of state-contingent inflation in isolation.

The key is that as a function of inflation volatility, the costs of resource misallocation are bounded. This is due to the nature of the sticky price firm's pricing decision. In sticky price models with flexible wages, marginal costs are extremely responsive to monetary shocks (see for instance Chari et al., 2000). This generates an asymmetry in the sticky price firm's profit function. Large inflationary shocks drive down the firm's relative price, so that demand is high, while driving up marginal costs. This leads to negative profits. On the other hand, large deflationary shocks

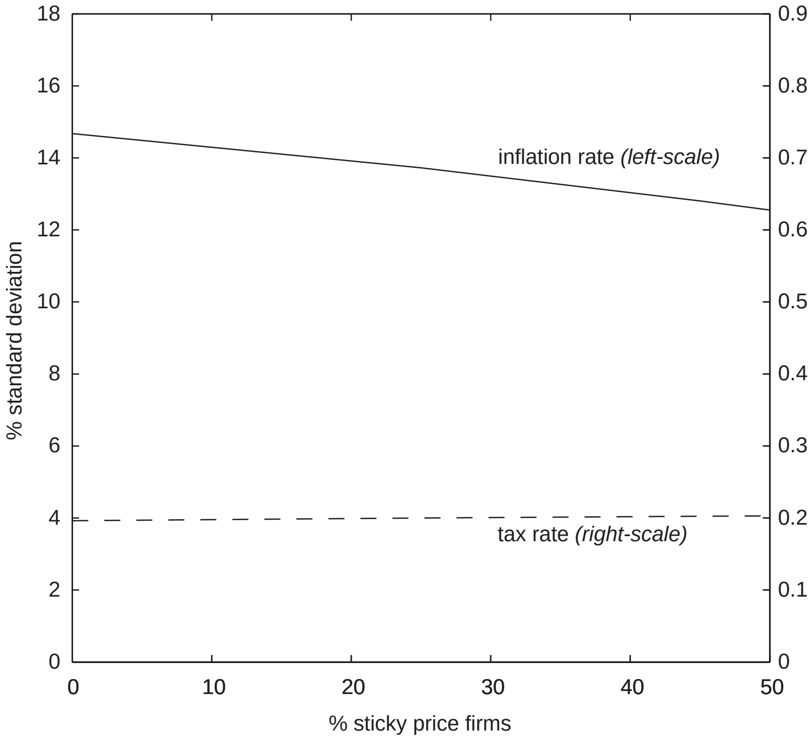

Fig. 6. Optimal inflation and tax rate volatility for large government spending shocks. Measured as percentage standard deviation for various degrees of price rigidity.

drive up the firm's relative price so that at worst, demand and profits are zero. In the face of volatile inflation, the sticky price firm hedges against the possibility of earning negative profits. In all periods, the sticky price firm prices as if government spending and inflation will be high in the following period.

This pricing behavior implies that in response to an ex-post inflation (when government spending is high), the ratio $L_s/L_f \simeq 1$, causing relatively little labor misallocation. However, when there is an ex-post deflation (when government spending is low) the sticky price firm's price is set 'too high', so that $L_s/L_f < 1$. As the volatility of government spending and inflation increase, the costs of resource misallocation increase as $L_s/L_f \to 0$ in deflation states. But since the misallocation cost is finite at $L_s/L_f = 0$ (see Fig. 1), the cost of adopting a state-contingent inflation policy is bounded above.

However, for increasingly volatile government spending processes, the benefits of state-contingent inflation are strictly increasing. This can be shown by comparing welfare across Ramsey equilibria in the following extreme cases: (i) a model with state-contingent real debt (as in Lucas and Stokey, 1983), and (ii) a model with non-state-contingent real debt (as in Aiyagari et al., 2002). In case (i), optimal policy sets

$\mathscr{PV}(\underline{s}^t) > \mathscr{PV}(\bar{s}^t)$ so that tax distortions are smoothed across states of nature, whereas in case (ii), policy is constrained in requiring $\mathscr{PV}(\underline{s}^t) = \mathscr{PV}(\bar{s}^t)$ so that only intertemporal tax smoothing is possible (see Section 5). In Siu (2002), I show that the cost of foregoing cross-state tax smoothing is increasing and convex in the volatility of the shock process. Hence, as government spending shocks grow, the benefits of state-contingent inflation increase at an increasing rate; at the same time, the costs of state-contingent inflation due to resource misallocation, though initially increasing, are eventually bounded. Combining these features, it is not surprising that for environments with sufficiently large shocks, optimal policy prescribes volatile inflation despite the presence of sticky prices.[19]

8. Conclusion

This paper characterizes optimal fiscal and monetary policy with sticky price setting in intermediate goods markets. With sticky prices, a benevolent government must balance the shock absorbing benefits of state-contingent inflation against its resource misallocation costs. The results of this study extend those found in the literature in a number of ways.

With government spending calibrated to post-war data, the Ramsey solution prescribes essentially constant deflation, even when the fraction of sticky price firms is small. Hence, responses in the real value of inherited government liabilities are largely attenuated. Instead, tax distortions can essentially be characterized as being smoothed over time. Persistent spells of high spending are accompanied by increasing tax collection and the accumulation of debt; spells of low spending by falling taxes and the reduction of debt. This imparts a high degree of persistence in tax rates and real debt holdings, regardless of the persistence in the underlying shock process. In summary, the extreme volatility in optimal inflation rates described in the optimal policy literature, at least for post-war calibrated shocks, is sensitive to small departures from the assumption of flexible price setting. However, for volatile government spending processes, inflation volatility is retained as a policy prescription despite the presence of sticky prices. Ramsey policy tolerates resource misallocation in favor of cross-state tax smoothing achieved through ex-post movements in the price level. The implications of this result for welfare experience associated with historical war episodes is still to be determined.

Appendix A. The flexible price model

A.1. Preliminaries

Here, I characterize competitive equilibrium for the cash–credit good model with flexible prices. The first set of equilibrium conditions are the household FONCs, the

[19] See Siu (2002) for further discussion and quantitative results for the large shock case.

household's budget constraint, the cash-in-advance constraint, and the government's budget constraint; these are identical to those presented in Section 2. The final good and intermediate good production functions are identical as well.

Since there are no sticky price firms, $Y_i(s^t) = \bar{Y}(s^t)$ for all i in a symmetric equilibrium. Also, $Y(s^t) = \bar{Y}(s^t)$ and $P_i(s^t) = P(s^t)$ for all i. Imposing labor market clearing, $\bar{Y}(s^t) = l(s^t)^{\alpha}$. From the intermediate good firm's FONC, $W(s^t) = (\alpha/\mu)P(s^t)l(s^t)^{\alpha-1}$. Clearing in the final goods market is satisfied by Walras' Law.

Proposition 6. *A symmetric equilibrium is characterized in primal form as an allocation* $\{c_1(s^t), c_2(s^t), l(s^t)\}$ *that satisfies the following three constraints*:

$$U_1(s^t) \geqslant U_2(s^t), \tag{A.1}$$

$$c_1(s^t) + c_2(s^t) + g(s^t) = l(s^t)^{\alpha}, \tag{A.2}$$

which hold for all s^t, *and*

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t)\bar{C}(s^t) = U_1(s^0)a_0, \tag{A.3}$$

where

$$\bar{C}(s^t) = U_1(s^t)c_1(s^t) + U_2(s^t)c_2(s^t) + \frac{\mu}{\alpha} U_l(s^t)l(s^t). \tag{A.4}$$

Furthermore, *given allocations which satisfy these constraints*, *it is possible to construct all of the remaining equilibrium real allocation*, *price and policy variables*.

Proof. The constraint $U_1(s^t) \geqslant U_2(s^t)$ guarantees no arbitrage. The second constraint is the aggregate resource constraint. These are obtained through substitution. To obtain the implementability constraint take (3), multiply by $\beta^t\pi(s^t)U_1(s^t)/P(s^t)$, and sum over all s^t and t. Using (5)–(8) and the transversality condition on real bonds, this simplifies to

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t)\left\{ \bar{C}(s^t) + U_l(s^t)\left[\frac{l(s^t)^{\alpha}}{w(s^t)} - \frac{\mu}{\alpha} l(s^t)\right]\right\} = U_1(s^0)a_0. \tag{A.5}$$

Finally, use the intermediate good firm's FONC to obtain (A.3).

With sequences $\{c_1(s^t), c_2(s^t), l(s^t)\}$ that satisfy (A.1)–(A.3), construct the remaining equilibrium objects at s^t as follows. Output, real balances and the gross nominal return are given by $\bar{Y}(s^t) = l(s^t)^{\alpha}$, $M(s^t)/P(s^t) = c_1(s^t)$, and $R(s^t) = U_1(s^t)/U_2(s^t)$. The real wage rate is

$$w(s^t) = \frac{\alpha}{\mu} l(s^t)^{\alpha-1}, \tag{A.6}$$

and the tax rate is

$$\tau(s^t) = 1 + \frac{U_l(s^t)}{U_2(s^t)w(s^t)}. \tag{A.7}$$

Real bond holdings and the gross rate of inflation are given by

$$b(s^t) = \sum_{r=t+1}^{\infty} \sum_{s^r|s^t} \beta^{r-t} \pi(s^r|s^t) \frac{\bar{C}(s^r)}{U_1(s^t)} + \frac{U_2(s^t)}{U_1(s^t)} c_2(s^t) + \frac{\mu}{\alpha} \frac{U_l(s^t)}{U_1(s^t)} l(s^t), \tag{A.8}$$

and

$$\frac{P(s^{t+1})}{P(s^t)} = \frac{R(s^t)b(s^t) + (1-\tau(s^t))\bar{Y}(s^t) - c_2(s^t)}{[c_1(s^{t+1}) + b(s^{t+1})]}, \tag{A.9}$$

respectively. □

This is the natural simplification of the primal form for the sticky price economy. In particular, without sticky prices, constraint (28) is irrelevant. Symmetry requires $L_i(s^t) = l(s^t)$ for all i, so that $\Lambda(s^t) \equiv (\mu/\alpha) l(s^t)$ in (26). Since $h(s^{t+1}) \equiv 0$ in (27), this constraint holds trivially.

A.2. When is the Friedman Rule optimal?

Given this result, it is easy to prove that the Friedman Rule is optimal in the flexible price model.

Proof. Consider maximizing the household's expected lifetime utility subject to (A.2) and (A.3), leaving the rate of return, $R(s^t)$, unconstrained. Equate the FONCs with respect to $c_1(s^t)$ and $c_2(s^t)$ for $t \geqslant 1$, and simplify to obtain

$$[U_1(s^t) - U_2(s^t)](1 + \lambda\chi(s^t)) = 0, \tag{A.10}$$

where λ is the Lagrange multiplier associated with (A.3), and $\chi = C_1/U_1 = C_2/U_2$. This last equality follows from the fact that preferences satisfy (2). Therefore, it must be that $U_1(s^t) = U_2(s^t)$, $t \geqslant 1$. It is also possible to show that $U_1(s^0) = U_2(s^0)$ is constrained optimal (so that $\delta(s^0) > 0$) for $a_0 > 0$. Therefore, the Friedman Rule is optimal. □

Indeed, this result holds for a more general class of utility functions than specified in condition (2). As long as preferences are homothetic in cash and credit good consumption, and weakly separable in leisure, optimality of the Friedman Rule is maintained (see Chari and Kehoe, 1999, for Friedman Rule results in a variety of monetary, flexible price models).

Note also that this result depends crucially upon labor and profit income being taxed at the uniform rate, $\tau(s^t)$. If the model is modified so that the tax rate on profits is zero, the Friedman Rule is no longer optimal. To see this, modify the model in this manner and derive the primal representation. Equilibrium is characterized by the same no arbitrage and aggregate resource constraints, but the implementability constraint becomes

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \tilde{C}(s^t) = U_1(s^0) a_0, \tag{A.11}$$

where

$$\tilde{C}(s^t) = U_1(s^t)c_1(s^t) + U_2(s^t)\left[c_2(s^t) - \left(1 - \frac{\alpha}{\mu}\right)l(s^t)^{\alpha}\right] + U_l(s^t)l(s^t). \tag{A.12}$$

Inspection of the Ramsey FONCs with this set of constraints reveals that the Friedman Rule is not optimal. In fact, in the Ramsey equilibrium, $U_1(s^t) > U_2(s^t)$ for all $s^t,\ t \geqslant 1$, so that the nominal interest rate should be strictly positive.

To gain intuition, note that the implementability constraint and aggregate resource constraint of this modified economy are equivalent to those derived from a repeated sequence of static, real barter economies. This economy features: a production technology with a linear transformation frontier between c_1 and c_2; distinct consumption tax rates; no income taxes; and an untaxed endowment of c_2 which, in equilibrium, is equal to $(1 - \alpha/\mu)l^{\alpha}$. Because of the endowment, optimality requires $U_1 > U_2$. Since the law of one price dictates that c_1 and c_2 are sold at a uniform price, the government levies a higher tax rate on c_1 to induce the optimum. In the context of the cash–credit good model, this is achieved through a positive nominal interest rate which acts as a tax on the cash good.

Hence, the fact that $R(s^t) > 1$ is optimal can be understood as an exception to the uniform commodity taxation rule. Despite preferences that are homothetic in c_1 and c_2, and weakly separable in leisure, optimal consumption tax rates are not equal when there is an untaxed endowment of one good (see Chari and Kehoe, 1999). In the cash–credit good model, the presence of untaxed profit income acts as a wealth endowment denominated in the credit good, since profit is transformed into credit one-for-one.

Appendix B. Derivations of cross-state constraints

B.1. Proof of Proposition 5

Proof. For convenience, I reproduce condition (28) as

$$A(\underline{s}^t) \sum_{r=t}^{\infty} \sum_{s^r|\underline{s}^t} \beta^{r-t}\pi(s^r|\underline{s}^t)\frac{C(s^r)}{U_1(\underline{s}^t)} = A(\bar{s}^t) \sum_{r=t}^{\infty} \sum_{s^r|\bar{s}^t} \beta^{r-t}\pi(s^r|\bar{s}^t)\frac{C(s^r)}{U_1(\bar{s}^t)}, \tag{B.1}$$

for $\underline{s}^t$ and $\bar{s}^t$ following s^{t-1}.

The term

$$\sum_{r=t}^{\infty} \sum_{s^r|s^t} \beta^{r-t}\pi(s^r|s^t)\frac{C(s^r)}{U_1(s^t)}, \tag{B.2}$$

is the present value of real government budget surpluses from state s^t onward. To see this, consider the expression:

$$\frac{U_1(s^t)}{P(s^t)}[M(s^t) + B(s^t)]. \tag{B.3}$$

Manipulating this as in the proof of Proposition 2 and dividing by $U_1(s^t)$ produces (B.2). However, (B.2) can be written in an alternative form. Again, begin with (B.3);

use the government's date r budget constraint, multiply by $\beta^r \pi(s^r \mid s^t) U_1(s^r)/P(s^r)$, and sum over states s^r following s^t and dates $r \geqslant t+1$ to get:

$$\sum_{r=t}^{\infty} \sum_{s^r|s^t} \beta^{r-t} \pi(s^r|s^t) U_1(s^r) \left[\frac{\tau(s^r) Y(s^r) - g(s^r)}{R(s^r)} + \frac{M(s^r)}{P(s^r)} \left(\frac{R(s^r) - 1}{R(s^r)} \right) \right]. \tag{B.4}$$

Divide by $U_1(s^t)$ to obtain the expression for $\mathscr{PV}(s^t)$. Hence, (B.2) is equivalent to $\mathscr{PV}(s^t)$, and the sticky price constraint can be rewritten as

$$A(\underline{s}^t)\mathscr{PV}(\underline{s}^t) = A(\bar{s}^t)\mathscr{PV}(\bar{s}^t), \tag{B.5}$$

for $\bar{s}^t$ and $\underline{s}^t$ following s^{t-1}. □

B.2. The cross-state constraint with incomplete markets

Here I describe a real economy with non-state-contingent debt and derive the cross-state constraint imposed on the Ramsey problem. This restriction is simply a rewriting of the *measurability constraint* of Aiyagari et al. (2002) for the simple case in which government spending takes on two possible values, $\{\underline{g}, \bar{g}\}$. The presentation follows closely that of Chari and Kehoe (1999) and Aiyagari et al. (2002).

The representative household maximizes utility derived from consumption, c, and leisure, $1 - l$. In each period, the household is subject to the following budget constraint:

$$c(s^t) + \tilde{b}(s^t) = (1 - \tau(s^t)) l(s^t) + \tilde{R}(s^{t-1}) \tilde{b}(s^{t-1}), \quad \forall s^t, \tag{B.6}$$

where $\tau(s^t)$ is the tax rate, and $\tilde{b}(s^t)$ are holdings of one-period real bonds. These mature at the beginning of period $t+1$, and earn a non-state-contingent real return of $\tilde{R}(s^t)$. Production is constant returns to labor, generating the following aggregate resource constraint:

$$c(s^t) + g(s^t) = l(s^t), \quad \forall s^t. \tag{B.7}$$

The government sets the tax rate and issues real bonds to satisfy its budget constraint:

$$\tilde{b}(s^t) + \tau(s^t) l(s^t) = \tilde{R}(s^{t-1}) \tilde{b}(s^{t-1}) + g(s^t), \quad \forall s^t. \tag{B.8}$$

To derive the cross-state constraint, take (B.8) and multiply by the marginal utility of consumption, $U_c(s^t)$, to get

$$U_c(s^t)[\tau(s^t) l(s^t) - g(s^t) + \tilde{b}(s^t)] = U_c(s^t) \tilde{R}(s^{t-1}) \tilde{b}(s^{t-1}). \tag{B.9}$$

Add to this the government's date r budget constraint, multiplied by the term $\beta^{r-t} \pi(s^r|s^t) U_c(s^r)$, for all s^r following s^t and $r \geqslant t+1$. Using the household's intertemporal FONC, $U_c(s^t) = \beta \tilde{R}(s^t) \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) U_c(s^{t+1})$, produces

$$\sum_{r=t}^{\infty} \sum_{s^r|s^t} \beta^{r-t} \pi(s^r|s^t) U_c(s^r)[\tau(s^r) l(s^r) - g(s^r)] = U_c(s^t) \tilde{R}(s^{t-1}) \tilde{b}(s^{t-1}). \tag{B.10}$$

The summation term in (B.10) is the present (utility) value of real government surpluses. Defining $\mathscr{PV}(s^t)$ as the left-hand side of (B.10) divided by $U_c(s^t)$, the expression becomes $\mathscr{PV}(s^t) = \tilde{R}(s^{t-1}) \tilde{b}(s^{t-1})$. Note that $\tilde{R}(s^{t-1}) \tilde{b}(s^{t-1})$ is known at state s^{t-1} and must be identical across histories, s^t. Denote these as $\underline{s}^t$ and $\bar{s}^t$. Hence, the cross-state constraint imposed by non-state-contingent real returns can be expressed as $\mathscr{PV}(\underline{s}^t) = \mathscr{PV}(\bar{s}^t)$, which is identical to (46).

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